Lesson Title **1.5.4 Inverses & Determinants with Matrices** Algebra 2 Date ____________

Determinant:

- is a **value** associated with a square matrix.
- Notated by straight vertical brackets ie. |A|
- Also notated on by det A (this is how your calculator shows it)

The formula for the determinant of a 2x2 is:

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a(d) = b(c)$

TASK 1: Find the determinant of each matrix.

a) $\begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix}$

$1(8)-4(5) = 8-20$

$= \boxed{-12}$

b. $\begin{vmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{vmatrix}$

$= \frac{1}{2}(2) - (-1)(-\frac{1}{2})$

$= 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

Determinant:

When you find the value of the determinant of a 3x3 you use the Expansion by Minors method using diagonals.

$$M = \begin{bmatrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{bmatrix}$$

$$\det M = \begin{vmatrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{vmatrix}, \text{ so write } \left|\begin{array}{ccc|cc} 2 & 4 & 1 & 2 & 4 \\ 5 & 2 & 3 & 5 & 2 \\ 1 & 4 & 8 & 1 & 4 \end{array}\right.$$

Rewrite the first two columns at the right side of the determinant.

Add the sum of the products of the red diagonals. Then subtract the sum of the blue diagonals.

$$\det \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \left|\begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}\right. \quad a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$$

TASK 2: Find the determinant of $\begin{bmatrix} 2 & -3 & 4 \\ 5 & 1 & -2 \\ 10 & 3 & -1 \end{bmatrix}$ $\begin{matrix} 2 & -3 \\ 5 & 1 \\ 10 & 3 \end{matrix}$

$= (2)(1)(-1) + (-3)(-2)(10) + (4)(5)(3) - (10)(1)(4) - (3)(-2)(2) - (-1)(5)(-3)$

$= -2 + 60 + 60 - 40 + 12 - 15$

$= \boxed{75}$

When can I use determinants?

A triangle has vertices as (1, 2); (3, -4); and (-2, 3). Find the area of the triangle.

You could try to work from a drawing of the triangle, but this can get very complicated. Instead, I put the vertices of the triangle into a determinant, with the x*-values being the first column/row*, the corresponding y-values being the second column/row, and the *third row/column all filled with 1's*, like this:

TASK 3:

$$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 1 \\ -2 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & -4 \\ -2 & 3 \end{matrix}$$

$= \frac{1}{2}|-4 - 4 + 9 - 8 - 3 - 6|$

$= \frac{1}{2}|-16|$

$= \frac{1}{2}(16)$

$= \boxed{8u^2}$

*****REMEMBER WE CANNPT DIVIDE MATRICES.*****

Instead of dividing we multiply by the **INVERSE**. The inverse involves the determinant.

Formula for the inverse:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

TASK 4: Find the inverse of the matrix, if it is defined. If not, explain why.

a) $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ $4(1) - 2(3)$ $4 - 6 = -2$

$A^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}}$

b) $B = \begin{bmatrix} 4 & -3 \\ -\frac{1}{3} & \frac{1}{4} \end{bmatrix}$

$= 4(\frac{1}{4}) - (-\frac{1}{3})(-3)$

$= 1 - 1 = 0$

no inverse, $\det(B) = 0$

Still need help with: