Lesson Title **1.5.5 Cramer's Rule & Solving Systems with Matrices** Algebra 2 Date ____________

Solving Systems using Matrices:

Remember the solution to a system can happen three ways:

- one solution (the point of intersection (x, y) or (x, y, z)
- no solution (parallel lines or parallel planes)
- infinitely many solutions (coinciding lines or coinciding planes)

TASK 1: Solve the system of equations using matrices. Graph to check on your calculator.

$$x + 2y = 5$$
$$3x + 5y = 14$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 1(5) - 2(3) = 5 - 6 = \boxed{-1}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

$$\frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \boxed{(3,1)}$$

Steps:

a) Rewrite your system of equations into a matrix equation

$$\begin{matrix} 2x + 3y = 11 \\ x + 2y = 6 \end{matrix} \quad \leftrightarrow \quad \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

b) Normally we would divide by the coefficient to solve for our variable, but dividing is not possible with matrices, so we multiply by the inverse.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

c) To find the inverse remember you find the determinant first. Then you use the formula for the inverse.

$$\det\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = (2)(2) - (1)(3) = 1$$

$$\text{inverse: } \frac{1}{1}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

d) Once you multiply the inverse coefficient matrix by your answer matrix you will have your variable matrix but with numbers. Do not forget to write your answer as a coordinate.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 11 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \rightarrow (4, 1)$$

TASK 2: Solve the system of equations using matrices. Graph to check on your calculator.

$$5a + 3b = 7$$
$$3a + 2b = 5$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} = 5(2) - 3(3) = 10 - 9 = \boxed{1}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$\frac{1}{1}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \Rightarrow \boxed{(-1,4)}$$

$$x + 2y = 5$$
$$2x + 4y = 8$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 1(4) - 2(2) = 4 - 4 = 0$$

no det = no inverse

So... no sol. or infinite sol.

same slopes of $-\frac{1}{2}$ means parallel lines

$\boxed{\text{no solution}}$

Cramer's Rule: process used to solve a system of equations

$$2x + y = 10$$
$$3x - 2y = 8$$

1. *Write your system as a matrix equation.*

$$\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

2. *Find "d" the value of the determinant for the first matrix in your equation created from all four coefficients.*

$$d = \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} = 2(-2) - 3(1) = -7$$

3. *Find "d_1" the value of a new determinant created by replacing the first column in your coefficient matrix with the column from your answer matrix.*

$$d_1 = \begin{vmatrix} 10 & 1 \\ 8 & -2 \end{vmatrix} = 10(-2) - 8(1) = -28$$

4. *Find "d_2" the value of a new determinant created by replacing the second column in your coefficient matrix with the column from your answer matrix.*

$$d_2 = \begin{vmatrix} 2 & 10 \\ 3 & 8 \end{vmatrix} = 2(8) - 3(10) = -14$$

5. *Your solution to a system is the point of intersection. In this example your solution is an ordered pair. The x and y values comes from the formula below.*

$$x = \frac{d_1}{d} = \frac{-28}{-7} = 4 \text{ and } y = \frac{d_2}{d} = \frac{-14}{-7} = 2$$

Solution is (4, 2)

6. *If there was a third variable then your solution would be an ordered triple, (x, y, z) and you would have a third determinant. You would repeat the same process; the answer column would replace the third column in the coefficient matrix.*

$$z = \frac{d_3}{d}$$

Still need help with:

TASK 1: Solve the system of equations using Cramer's Rule. Graph to check on your calculator.

$$x + 2y = 5$$
$$3x + 5y = 14$$

$d = \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 1(5) - 3(2) = \boxed{-1}$

$d_1 = \begin{vmatrix} 5 & 2 \\ 14 & 5 \end{vmatrix} = 5(5) - 14(2) = \boxed{-3}$

$d_2 = \begin{vmatrix} 1 & 5 \\ 3 & 14 \end{vmatrix} = 1(14) - 3(5) = \boxed{-1}$

$x = \frac{d_1}{d} = \frac{-3}{-1} = 3$

$y = \frac{d_2}{d} = \frac{-1}{-1} = 1$

$\boxed{(3,1)}$

TASK 2: Solve the system of equations using Cramer's Rule. Graph to check on your calculator.

$$5a + 3b = 7$$
$$3a + 2b = 5$$

$d = \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} = 5(2) - 3(3) = \boxed{1}$

$d_1 = \begin{vmatrix} 7 & 3 \\ 5 & 2 \end{vmatrix} = 7(2) - 5(3) = \boxed{-1}$

$d_2 = \begin{vmatrix} 5 & 7 \\ 3 & 5 \end{vmatrix} = 5(5) - 3(7) = \boxed{4}$

$x = \frac{d_1}{d} = \frac{-1}{1} = \boxed{-1}$

$y = \frac{d_2}{d} = \frac{4}{1} = \boxed{4}$

$\boxed{(-1,4)}$

TASK 3: Solve the system of equations using Cramer's Rule. Graph to check on your calculator.

$$x + 2y = 5$$
$$2x + 4y = 8$$

$d = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 1(4) - 2(2) = \boxed{0}$

// $\overset{5}{\leftrightarrow}$

$m = -\frac{1}{2}$

no det! no inverse

no intersection ⇒ $\boxed{\text{no solution}}$