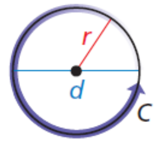
11.1 Circumference and Arc Length

Essential Question
How can you find the length of a circular arc?

Core Concept

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

Example 1: Find each indicated measure.

circumference of a circle with a radius of 11 inches

$C = 2\pi r = 2\pi(11) = \boxed{22\pi \text{ in}} \approx \boxed{69.115 \text{ in}}$

Practice:

1. radius of a circle with a circumference of 4 millimeters

$C = 2\pi r = 4 = 2\pi r$ $\quad \boxed{\frac{2}{\pi} \text{ mm} = r}$ $\quad \boxed{\approx .637 \text{ mm}}$

2. circumference of a circle with diameter of 6 centimeters. Leave answer exact

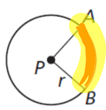
$C = \pi d \Rightarrow C = \pi(6)$ $\quad \boxed{6\pi \text{ cm}}$

Core Concept

Core Concept

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

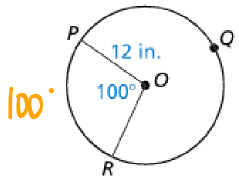


$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°}$, or

$\boxed{\text{Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r}$

Example 2: Find each indicated measure.

a. arc length of $\widehat{PR}$

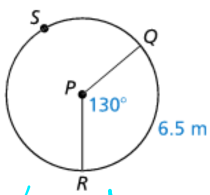


$= \left(\frac{100}{360}\right)(2\pi(12))$

$= \frac{5}{18}(24\pi)$

$= \boxed{\frac{20\pi}{3} \text{ in}}$

b. circumference of $\odot P$

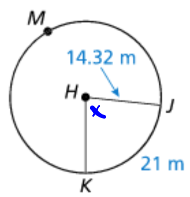


$6.5 = \left(\frac{130}{360}\right)C$

$2340 = 130\,C$

$\boxed{18 \text{ m} = C}$

c. $m\widehat{JK}$



$21 = \left(\frac{x}{360}\right)(2\pi(14.32))$

$7560 = (28.64\pi)x$

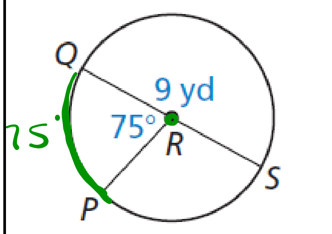
$\boxed{263.966\pi \approx x}$

Core Concept

Practice:

Find the indicated measure.

3. arc length of $\overset{\frown}{PQ}$

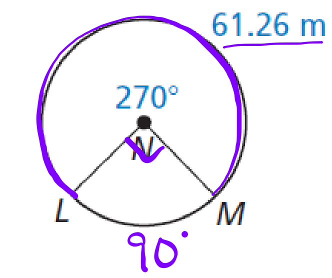


$= \left(\frac{75}{360}\right)(\pi 9)$

$= \frac{675\pi}{360}$

$= \boxed{\frac{15\pi}{8} \text{ yd}}$

4. circumference of ⊙N

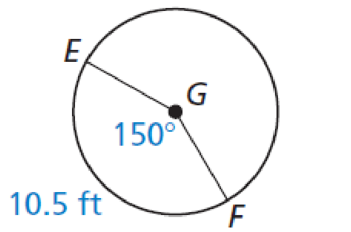


$61.26 = \frac{270}{360}(C)$

$22{,}053.6 = 270C$

$\boxed{81.68 \text{ m} = C}$

5. radius of ⊙G



$10.5 = \frac{150°}{360°}(2\pi r)$

$3780 = (300\pi)r$

$\boxed{\frac{12.6 \text{ ft}}{\pi} = r}$

Monitoring Progress 3-5

Example 3: The radius of a wheel on a toy truck is 4 inches. To the nearest foot, how far does the tire travel when it makes 7 revolutions?

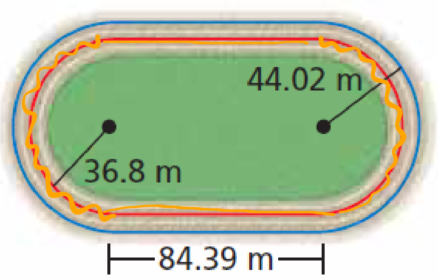
$C = 4(2\pi)$

$= 8\pi$

$= 56\pi \approx 175.929$

$\boxed{176 \text{ ft}}$

Example 4: The curves at the ends of the track shown are 180° arcs of circles. The radius of the arc for a runner on the red path shown is 36.8 meters. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.



$= 2(84.39) + 2(2\pi(36.8))$

$= 168.78 + 147.2\pi$

≈ 631.222

$\boxed{631.2 \text{ m}}$

Example 3

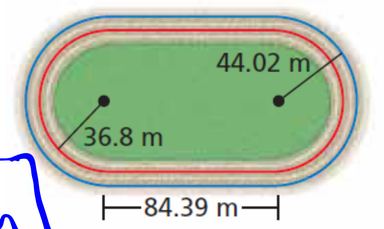
Practice

6. A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

$500 \equiv \pi r\ (28\pi)$

$\boxed{56.1 \text{ revolutions}}$

7. In Example 4, the radius of the arc for a runner on the blue path is 44.02 meters, as shown in the diagram. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.



$= 2(89.39) + 2(44.02\pi(2))$

$= 178.78 + 176.08\pi$

≈ 731.952

$\boxed{732 \text{ m}}$

Monitoring Progress 6-7

Core Concept

Converting between Degrees and Radians

Degrees to radians

Multiply degree measure by

$\frac{2\pi \text{ radians}}{360^\circ}$, or $\frac{\pi \text{ radians}}{180^\circ}$.

Radians to degrees

Multiply radian measure by

$\frac{360^\circ}{2\pi \text{ radians}}$, or $\frac{180^\circ}{\pi \text{ radians}}$.

Example 5: Convert the following.

a. Convert 30° to radians.

$\frac{30}{1} \cdot \frac{\pi r}{180} = \boxed{\frac{\pi}{6}}$

b. Convert $\frac{3\pi}{8}$ radians to degrees.

$\frac{3\cancel{\pi}}{8} \cdot \frac{180}{\cancel{\pi} r} = \frac{540\cancel{\pi}}{8\cancel{\pi}}$ $\boxed{67.5^\circ}$

Practice:

8. Convert 15° to radians.

$15 \cdot \frac{\pi}{180} = \frac{15\pi}{180}$

$= \boxed{\frac{\pi}{12}}$

9. Convert $\frac{4\pi}{3}$ radians to degrees.

$\frac{4\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}} = \boxed{240^\circ}$

Core Concept

HW: p. 598:

A: 11, 19 - 33 (o), 37, 43, 44

B: 1, 5, 7, 11-17(o), 23, 25, 27, 35, 37, 43, 44

C: 1 - 31 (o), 43, 44

ANSWERS:

44. 42 u^2

Mar 6-9:25 AM