

OBJECTIVE 2: Completing the Square

First it is all about finding the perfect "c". Complete the square using $\square = \left(\frac{b}{2}\right)^2$

TASK 1: Use the formula for completing the square above to fill the \square 's below.

a) $x^2 + 6x + \square$

$\square = 9$

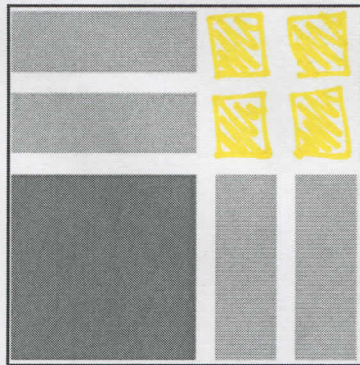
$\left(\frac{6}{2}\right)^2 = (3)^2 = 9 \quad (x+3)^2$

b) $x^2 - 5x + \square$

$\square = \frac{25}{4}$
 $\left(\frac{-5}{2}\right)^2 = \frac{25}{4} \quad \left(x - \frac{5}{2}\right)^2$

How many **yellow**s are needed to complete the square and balance the equation?

$x^2 + 4x + \square = -2 + \square$



STEPS:

- 1) If there is a "c" move it to the other side of the =.
- 2) Make sure "a" = 1, if not 1 then factor "a" out using GCF.
- 3) Divide the value of "b" by 2, square it, and add that value to both sides of the equation.
- 4) Write one side as a perfect square binomial and combine like terms on the other side, if necessary.
- 5) If "a" is not 1 then divide both sides by "a" to get the perfect square binomial alone.
- 6) Square root both sides of the equations, DO NOT FORGET the \pm in front of the square root.
- 7) Now subtract by your "h" to get x alone.
- 8) Simplify if necessary.
- 9) Check if time allows.

Task 2: Solve using completing the square.

a) $x^2 + 2x = 4$

$x^2 + 2x + \square = 4 + \square$

$(x+1)^2 = 5$

$x+1 = \pm\sqrt{5}$

$x = -1 \pm\sqrt{5}$

b) $x^2 + 4x = 3$

$x^2 + 4x + \square = 3 + \square$

$\sqrt{(x+2)^2} = \sqrt{7}$

$x+2 = \pm\sqrt{7}$

$x = -2 \pm\sqrt{7}$

c) $x^2 + 4x - 21 = 0$

$x^2 + 4x + \square = 21 + \square$

$\sqrt{(x+2)^2} = \sqrt{25}$

$x+2 = \pm 5$

$x = -2 \pm 5$

$x = -2 + 5 \quad x = -2 - 5$

$x = 3, -7$

$\square = \left(\frac{4}{2}\right)^2$

$\square = \left(\frac{2}{2}\right)^2$

$$\square = \left(\frac{4}{2}\right)^2 = (2)^2 = 4$$

Task 3: Solve using completing the square.

a) $x^2 - 7x - 1 = 0$

$$x^2 - 7x + \frac{49}{4} = 1 + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{4}{4} + \frac{49}{4}$$

$$\sqrt{\left(x - \frac{7}{2}\right)^2} = \sqrt{\frac{53}{4}}$$

$$x - \frac{7}{2} = \pm \sqrt{\frac{53}{4}}$$

$$x = \frac{7}{2} \pm \frac{\sqrt{53}}{2}$$

b) $2x^2 - 8x + 3 = 0$

$$2(x^2 - 4x + \frac{4}{2}) = -3 + 2(\frac{4}{2})$$

$$2(x-2)^2 = \frac{5}{2}$$

$$\sqrt{(x-2)^2} = \sqrt{\frac{5}{2}}$$

$$x-2 = \pm \sqrt{\frac{5}{2}}$$

$$x = 2 \pm \sqrt{\frac{5}{2}}$$

c) $x^2 - 3x + 1 = 0$

$$x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = -\frac{4}{4} + \frac{9}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{5}{4}}$$

$$x - \frac{3}{2} = \pm \sqrt{\frac{5}{4}}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$$

$$\square = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

Task 4: Solve using completing the square.

a) $3x^2 - 12x + 1 = 0$

$$3(x^2 - 4x + \frac{4}{3}) = -1 + 3(\frac{4}{3})$$

$$3(x-2)^2 = \frac{11}{3}$$

$$\sqrt{(x-2)^2} = \sqrt{\frac{11}{3}}$$

$$x-2 = \pm \sqrt{\frac{11}{3}}$$

$$x = 2 \pm \sqrt{\frac{11}{3}}$$

$$\left(\frac{-3}{2}\right)^2 = \left(\frac{9}{4}\right)$$

b) $3x^2 - 9x + 8 = 0$

$$3(x^2 - 3x + \frac{9}{4}) = -8 + 3(\frac{9}{4})$$

$$3(x - \frac{3}{2})^2 = -\frac{32}{4} + \frac{27}{4}$$

$$\frac{1}{3} \cdot 3(x - \frac{3}{2})^2 = \frac{-5}{4} \cdot \frac{1}{3}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-5}{12}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{-5}{12}}$$

c) $2x^2 - 5x + 7 = 0$

$$2(x^2 - \frac{5}{2}x + \frac{25}{16}) = -7 + 2(\frac{25}{16})$$

$$2(x - \frac{5}{4})^2 = -7 + \frac{50}{16}$$

$$2(x - \frac{5}{4})^2 = \frac{-112 + 50}{16}$$

$$2(x - \frac{5}{4})^2 = \frac{-62}{16}$$

$$(x - \frac{5}{4})^2 = \frac{-31}{8}$$

$$x - \frac{5}{4} = \pm \sqrt{\frac{-31}{8}}$$

$$x = \frac{5}{4} \pm \sqrt{\frac{-31}{8}}$$

$$\square = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$$

Reminders to myself about the Completing the Square:

- don't forget the \pm
- don't forget perfect squares: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169
- don't forget $a = 1$
- don't forget $\square = \left(\frac{b}{2}\right)^2$
- factoring is always $(x + \left(\frac{b}{2}\right))^2$

Still need help with:

$$\square = \left(\frac{-7}{2}\right)^2 = \frac{49}{4}$$

$$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$