

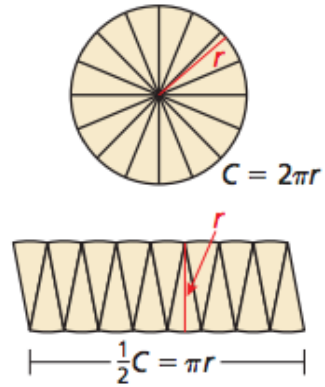
11.2 Area of Circles and Sectors

Essential Question How can you find the area of a sector of a circle?

Using the Formula for the Area of a Circle

You can divide a circle into congruent sections and rearrange the sections to form a figure that approximates a parallelogram. Increasing the number of congruent sections increases the figure's resemblance to a parallelogram.

The base of the parallelogram that the figure approaches is half of the circumference, so $b = \frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$. The height is the radius, so $h = r$. So, the area of the parallelogram is $A = bh = (\pi r)(r) = \pi r^2$.



Core Concept

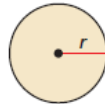
Core Concept

Area of a Circle

The area of a circle is

$$A = \pi r^2$$

where r is the radius of the circle.



Example 1: Find each indicated measure.

a. area of a circle with a radius of 8.5 inches

$$A = \pi (8.5)^2$$

$$= \pi (72.25)$$

$$= 72.25\pi \text{ in}^2$$

$$\approx 226.980 \text{ in}^2$$

Practice:

1. Find the area of a circle with a radius of 4.5 meters.

$$A = \pi (4.5)^2$$

$$= \pi (20.25) \text{ m}^2 \approx 63.617 \text{ m}^2$$

b. diameter of a circle with an area of 153.94 square feet

$$153.94 = \pi r^2$$

$$\sqrt{49.001} = \sqrt{r^2}$$

$$7.000 \approx r$$

$$d = 14 \text{ ft}$$

2. Find the radius of a circle with an area of 176.7 square feet.

$$176.7 = \pi r^2$$

$$\sqrt{56.245} = \sqrt{r^2}$$

$$7.4997 \approx r$$

$$r \approx 7.5 \text{ ft}$$

11.2 Area of Circles and Sectors with work

Using the Formula for Population Density

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

Example 2:

a. About 124,000 people live in a 2-mile radius of a city's post office. Find the population density in people per square mile.

$$\text{PD} = \frac{124,000}{\pi 2^2} \approx 9,867.606$$



The population density is about 9868 people per square mile.



b. A region with a 10-mile radius has a population density of about 869 people per square mile. Find the number of people who live in the region.



$$869 = \frac{X}{100\pi}$$

$$X \approx 273,004.402$$

The number of people who live in the region is about 273,004.

Practice:

3. About 58,000 people live in a region with a 2-mile radius. Find the population density in people per square mile.



$$\frac{58,000}{4\pi} \approx 45,553.093$$

45,553 people per square mile

4. A region with a 3-mile radius has a population density of about 1000 people per square mile. Find the number of people who live in the region.



$$\frac{1000}{1} = \frac{X}{9\pi}$$

$$X \approx 2827.433$$

2,827 people

11.2 Area of Circles and Sectors with work

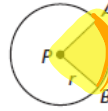
Core Concept

Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360° .

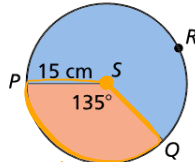
$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or}$$

$$\text{Area of sector } APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$



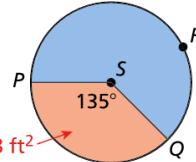
Example 3:

a. Find the areas of the sectors formed by $\angle PSQ$.



$$\begin{aligned} &= \frac{135}{360} (225\pi) \\ &= \frac{675\pi}{8} \text{ cm}^2 \\ &\approx 265.072 \text{ cm}^2 \end{aligned}$$

b. Find the area of $\odot S$.

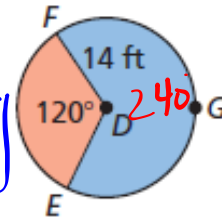


$$\begin{aligned} 48 &= \frac{135}{360} (A) \\ 17,280 &= 135A \\ \boxed{128\text{ft}^2} &= A \end{aligned}$$

Practice: Find the indicated measure.

5. area of red sector = $\frac{120}{360} (\pi(14)^2) = \frac{1}{3} (196\pi)$

$$\boxed{\frac{196\pi}{3} \text{ ft}^2}$$



6. area of blue sector

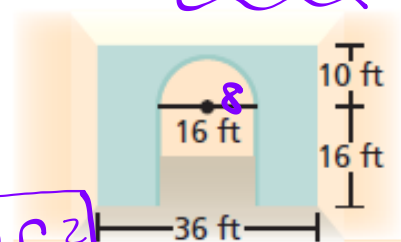
$$= \frac{240}{360} (196\pi) = \boxed{\frac{392\pi}{3} \text{ ft}^2}$$

7. A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?



$$\begin{aligned} &36(26) - (64\pi) - 256 \\ &936 - 201.062 - 256 \\ &\approx 478.938 \end{aligned}$$

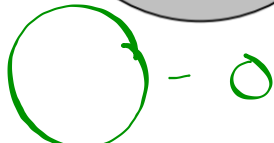
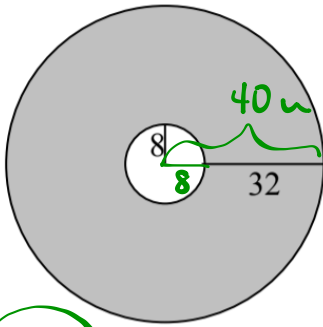
$$\boxed{479\text{ft}^2}$$



11.2 Area of Circles and Sectors with work

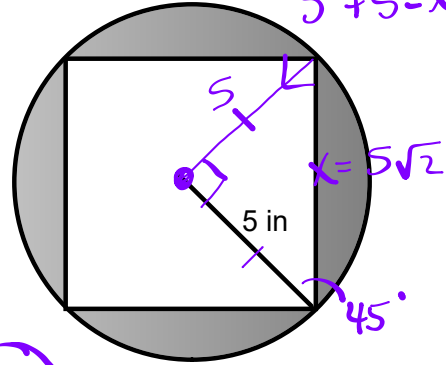
Example 4: Find the area of each shaded region.

a)



$$\begin{aligned}
 & (40)^2\pi - (8)^2\pi \\
 & 1600\pi - 64\pi \\
 & = \boxed{1536\pi \text{ u}^2} \\
 & \approx \boxed{4825.486 \text{ u}^2}
 \end{aligned}$$

b)



$$\begin{aligned}
 & (5)^2\pi - (5\sqrt{2})^2 \\
 & = \boxed{25\pi - 50 \text{ in}^2} \\
 & \approx \boxed{28.540 \text{ in}^2}
 \end{aligned}$$

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Practice 8: Find the shaded area.

Give exact answers AND approximations.

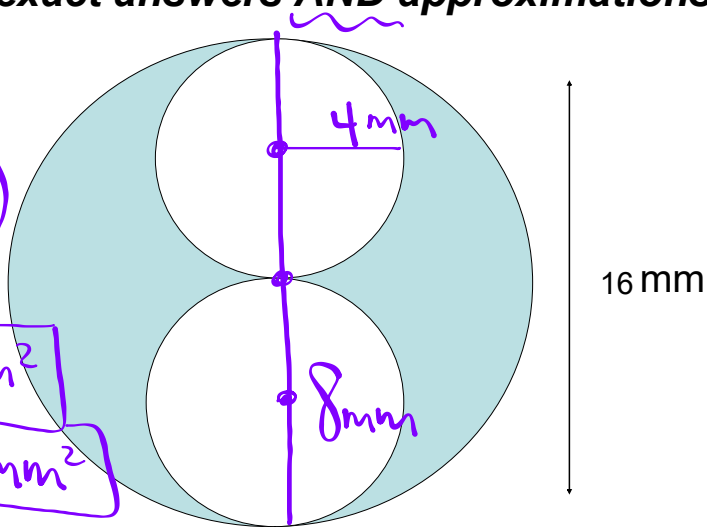


$$(16)^2\pi - 2(4^2\pi)$$

$$256\pi - 32\pi$$

$$= \boxed{224\pi \text{ mm}^2}$$

$$\approx \boxed{703.717 \text{ mm}^2}$$



Answer:

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11.2 Area of Circles and Sectors with work

HW: p. 606

A: 19, 21, 27, 29, 31, 35, 37, 42 - 45

B: 1, 3, 7, 9, 13, 17, 19, 21, 27, 29, 31, 42 - 45

C: 1 - 31 (o), 42 - 45

ANSWERS:

42. 54 in.^2

44. 58.5 in.^2

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Attachments

Area of a circle.gsp