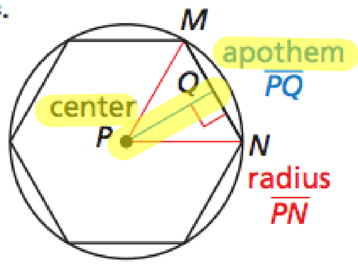
11.3 Area of Polygons

Finding Angle Measures in Regular Polygons



$\angle MPN$ is a central angle.

The diagram shows a regular polygon inscribed in a circle. The **center of a regular polygon** and the **radius of a regular polygon** are the center and the radius of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs. The word "apothem" refers to a segment as well as a length. For a given regular polygon, think of *an* apothem as a segment and *the* apothem as a length.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

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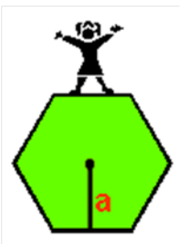
Area of a regular polygon



Finding Areas of Regular Polygons

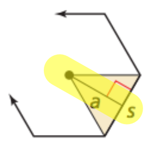
You can find the area of any regular n-gon by dividing it into congruent triangles.

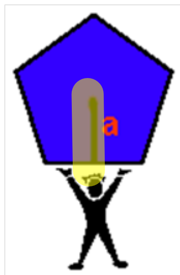
A = Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$ — Base of triangle is s and height of triangle is a. Number of triangles is n.

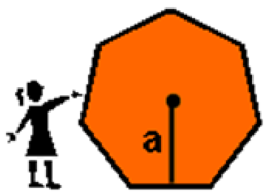
$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ — Commutative and Associative Properties of Multiplication

$= \frac{1}{2}a \cdot P$ — There are n congruent sides of length s, so perimeter P is $n \cdot s$.





$A = \frac{1}{2}ap$	Regular polygons have all sides of equal length . a = apothem p = perimeter



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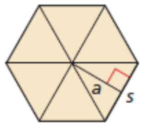
Core Concept

Area of a Regular Polygon

The area of a regular n-gon with side length s is one-half the product of the apothem a and the perimeter P.

$\boxed{A = \frac{1}{2}aP}$, or $A = \frac{1}{2}a \cdot ns$

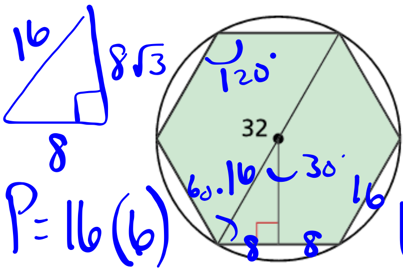


Example 0:

A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.

$\frac{(n-2)(180)}{n}$

$\frac{4(180)}{6} = 120^\circ$



$P = 16(6)$

$A = \frac{1}{2}(8\sqrt{3})(96)$

$\boxed{= 384\sqrt{3}u^2}$

$A = \frac{1}{2}aP = \frac{1}{2}(8\sqrt{3})(6 \cdot 16)$, or about

$\boxed{665.11 \text{ square units}}$

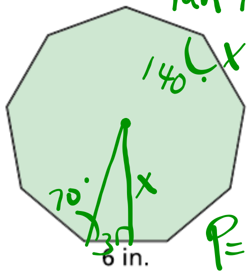
$\frac{7(180)}{9} = 140^\circ$

A mirror is in the shape of a regular nonagon with 6-inch sides. What is the area of the mirror?

9

$\tan 70^\circ = \frac{x}{3}$

$x = 3\tan 70$



$P = 6(9)$

$A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{3}{\tan 20^\circ}\right)(9 \cdot 6)$, or about

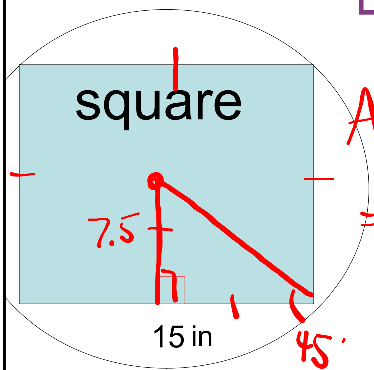
$\boxed{222.5 \text{ square inches}}$

$A = \frac{1}{2}(3\tan 70)(54)$

Example 1: Find the apothem and area for the figures below.

RECALL: degrees of an interior angle of a regular polygon.

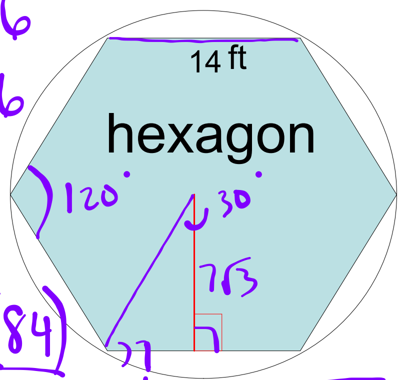
$$\boxed{\frac{(n-2)\,180^\circ}{n}} = \frac{4(180)}{6} = 120^\circ$$



$A = \frac{1}{2}(7.5)(60)$

$= \boxed{225 in^2}$

$P = 14 \cdot 6$



$A = \frac{1}{2}(7\sqrt{3})(84)$

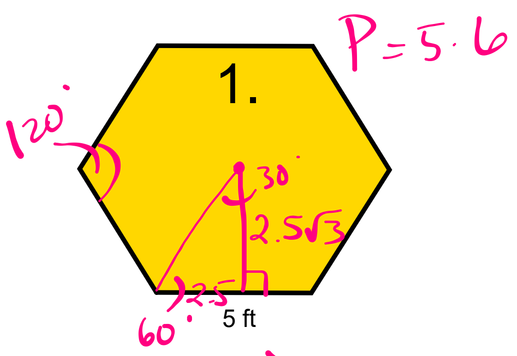
$= \boxed{294\sqrt{3} ft^2}$

$\approx \boxed{509.223 ft^2}$

a =
A =

a =
A =

Practice: Find the area of each regular polygon.

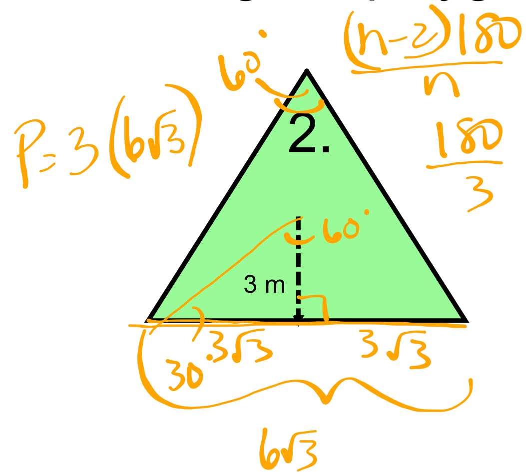


$A = \frac{1}{2}(2.5\sqrt{3})(30)$

$= \boxed{37.5\sqrt{3}\ ft^2}$

$\approx \boxed{64.952\ ft^2}$

Area =



$A = \frac{1}{2}(3)(18\sqrt{3})$

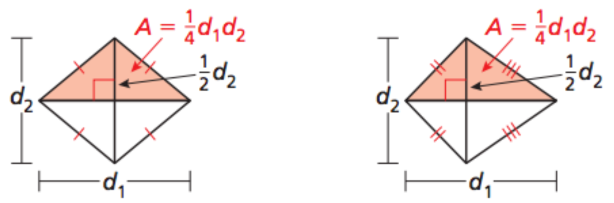
$= \boxed{27\sqrt{3}\ m^2} \approx \boxed{46.765\ m^2}$

Area =

Feb 15-1:41 PM

Finding Areas of Rhombuses and Kites

You can divide a rhombus or kite with diagonals d_1 and d_2 into two congruent triangles with base d_1, height $\frac{1}{2}d_2$, and area $\frac{1}{2}d_1\left(\frac{1}{2}d_2\right) = \frac{1}{4}d_1d_2$. So, the area of a rhombus or kite is $2\left(\frac{1}{4}d_1d_2\right) = \frac{1}{2}d_1d_2$.



Core Concept

Area of a Rhombus or Kite

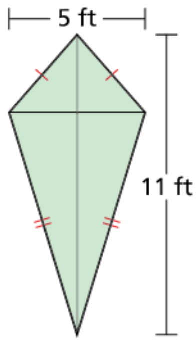
The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1d_2$.

Example 2:

Find the area of each rhombus or kite.

$A = \frac{1}{2} d_1 d_2$

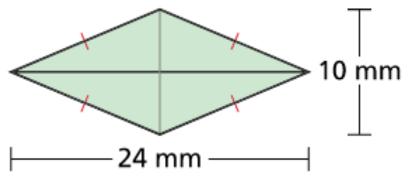
a.



$A = \frac{1}{2}(5)(11)$

$= \boxed{27.5 ft^2}$

b.



$A = \frac{1}{2}(10)(24)$

$= \boxed{120 mm^2}$

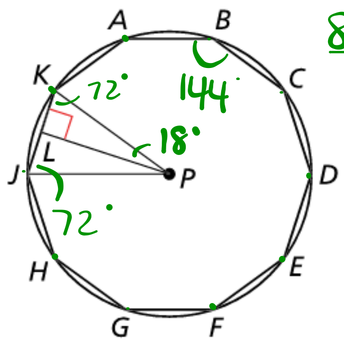
Example 3:

In the diagram, polygon *ABCDEFGHJK* is a regular decagon inscribed in ⊙*P*. Find each angle measure.

10 sides

$\frac{8(180)}{10}$

144



a. $m\angle KPJ = 36°$
b. $m\angle LPK = 18°$
c. $m\angle LJP = 72°$

Practice:

In the diagram, WXYZ is a square inscribed in ⊙P.

3. Identify the center, a radius, an apothem, and a central angle of the polygon.

Center P; $\overline{YP}$ or $\overline{XP}$; $\overline{PQ}$

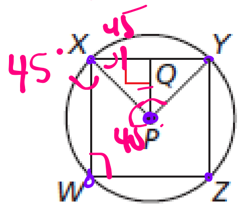
$\angle XPY$

4. Find m∠XPY, m∠XPQ, and m∠PXQ

$m\angle XPY = 90°$

$m\angle XPQ = 45°$

$m\angle PXQ = 45°$



HW: pg. 614

A: 25, 29, 31, 37, 39, 41, 47, 49

B: 1, 5, 9, 13, 17, 19, 25, 29, 31, 37

C: 1 - 31 (o)

Mar 7-6:44 AM