

# 11.3 Solving Equations by Using Quadratic Methods

## OBJECTIVE 1: Solving Equations That Are Quadratic in Form

Simplify first ALWAYS!

**Solving a Quadratic Equation**

**Step 1.** If the equation is in the form  $(ax + b)^2 = c$ , use the square root property and solve. If not, go to Step 2.

**Step 2.** Write the equation in standard form:  $ax^2 + bx + c = 0$ .

**Step 3.** Try to solve the equation by the factoring method. If not possible, go to Step 4.

**Step 4.** Solve the equation by the quadratic formula.

### Example 1: Solve: $x - \sqrt{x} - 6 = 0$

$$9 - \sqrt{9} - 6 = 0$$

$$4 - \sqrt{4} - 6 = -4$$

■

$$0$$

$$-4$$

$$(x-6)^2 = \sqrt{x}$$

$$(x-6)(x-6) = \sqrt{x}$$

$$x^2 - 12x + 36 = \sqrt{x}$$

$$x^2 - 13x + 36 = 0$$

$$(x-9)(x-4) = 0$$

$$x-9=0 \quad x-4=0$$

$$x=9, 4$$

$$x=9$$

### Practice 1: $x - \sqrt{x+1} - 5 = 0$

$$8 - \sqrt{8+1} - 5 = 0$$

$$3 - \sqrt{3+1} - 5 = -4$$

■

$$0$$

$$-4$$

$$(x-5)^2 = \sqrt{x+1}$$

$$x^2 - 10x + 25 = \sqrt{x+1}$$

$$x^2 - 11x + 24 = 0$$

$$(x-8)(x-3) = 0$$

$$x-8=0 \quad x-3=0$$

$$x=8 \quad x \neq 3$$

Example 2: Solve:  $\frac{3x}{x-2} - \frac{x+1}{x} = \frac{6}{x(x-2)}$

LCD:  $x(x-2)$   
 $a=2$   $b=1$   $c=-4$   
 $x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{2(2)}$   
 $x = \frac{-1 \pm \sqrt{33}}{4}$

$3x^2 - (x-2)(x+1) = 6$   $(1)^2 - 4(2)(-4)$   
33  
 $3x^2 - (x^2 - 2x + x - 2) = 6$   
 $3x^2 - (x^2 - x - 2) = 6$   
 $3x^2 - 1(x^2 - x - 2) = 6$   
 $3x^2 - x^2 + x + 2 - 6 = 0$   
 $2x^2 + x - 4 = 0$

Practice 2:  $\frac{5x}{x+1} - \frac{x+4}{x} = \frac{3}{x(x+1)}$

LCD:  $x(x+1)$   
 $a=4$   
 $b=-5$   
 $c=-7$

$5x^2 - (x+1)(x+4) = 3$   
 $5x^2 - (x^2 + x + 4x + 4) = 3$   
 $5x^2 - (x^2 + 5x + 4) - 3 = 0$   
 $5x^2 - x^2 - 5x - 4 - 3 = 0$   
 $4x^2 - 5x - 7 = 0$   
 $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-7)}}{2(4)}$   
 $x = \frac{5 \pm \sqrt{137}}{8}$

$(1)^2 - 4(2)(-4)$  33  
 $(-5)^2 - 4(4)(-7)$  137

Example 3: Solve:  $p^4 - 3p^2 - 4 = 0$

$(p^2 - 4)(p^2 + 1) = 0$   
 $(p+2)(p-2)(p^2 + 1) = 0$   
 $\frac{p+2=0}{p=-2}$      $\frac{p-2=0}{p=2}$      $\frac{p^2+1=0}{\sqrt{p^2} = \pm 1}$   
 $p = \pm i$

$(-2)^4 - 3(-2)^2 - 4$  0 ✓  
 $(2)^4 - 3(2)^2 - 4$  0 ✓

Practice 3:  $p^4 - 7p^2 - 144 = 0$

$(p^2 - 16)(p^2 + 9) = 0$   
 $(p+4)(p-4)(p^2 + 9) = 0$   
 $\frac{p+4=0}{p=-4}$      $\frac{p-4=0}{p=4}$      $\frac{p^2+9=0}{\sqrt{p^2} = \pm 3}$   
 $p = \pm 3i$

$(-4)^4 - 7(-4)^2 - 144$  0 ✓  
 $(4)^4 - 7(4)^2 - 144$  0 ✓

Helpful Hint

Example 3 can be solved using substitution also. Think of  $p^4 - 3p^2 - 4 = 0$  as

$$(p^2)^2 - 3p^2 - 4 = 0 \quad \text{Then let } x = p^2 \text{ and solve and substitute back.}$$

$$\downarrow \quad \downarrow \quad \text{The solutions will be the same}$$

$$x^2 - 3x - 4 = 0$$

✓ CONCEPT CHECK

- a. True or false? The maximum number of solutions that a quadratic equation can have is 2. ✓
- b. True or false? The maximum number of solutions that an equation in quadratic form can have is 2.

Example 4: Solve:  $(x - 3)^2 - 3(x - 3) - 4 = 0$

$y = x - 3$

$(7-3)^2 - 3(7-3) - 4 = 0$  ✓  
 $(2-3)^2 - 3(2-3) - 4 = 0$  ✓

$y^2 - 3y - 4 = 0$   
 $(y-4)(y+1) = 0$   
 $y-4=0 \quad y+1=0$   
 $y=4 \quad y=-1$   
 $x-3=4 \quad x-3=-1$   
 $x=7 \quad x=2$

Practice 4:  $(x + 2)^2 - 2(x + 2) - 3 = 0$

$y = x + 2$

$y^2 - 2y - 3 = 0$   
 $(y-3)(y+1) = 0$   
 $y-3=0 \quad y+1=0$   
 $y=3 \quad y=-1$   
 $x+2=3 \quad x+2=-1$   
 $x=1 \quad x=-3$

$(-3+2)^2 - 2(-3+2) - 3 = 0$  ✓  
 $(1+2)^2 - 2(1+2) - 3 = 0$  ✓

**OBJECTIVE 2:** Solving Problems That Lead to Quadratic Equations

Work problems are modeled by rational equations that simplify to be a quadratic equation.

**Example 6:** Finding Work Time

Together, an experienced word processor and an apprentice word processor can create a word document in 6 hours. Alone, the experienced word processor can create the document 2 hours faster than the apprentice word processor can. Find the time in which each person can create the word document alone.

	Total hours	Part of the job
Apprentice	x	$\frac{1}{x}$
Experienced	x - 2	$\frac{1}{x-2}$
Together	6	$\frac{1}{6}$

$LC: 6x(x-2)$   
 $13.1 \text{ hr}$   
 $1.1 \text{ hr}$   
 $6\left(\frac{1}{x} + \frac{1}{x-2}\right) = \frac{1}{6}$   
 $\frac{6(x-2)}{x} + \frac{6x}{x-2} = \frac{1}{6}$   
 $6x - 12 + 6x = x^2 - 2x$   
 $0 = x^2 - 14x + 12$   
 $x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(12)}}{2(1)} = \frac{14 \pm \sqrt{148}}{2} = \frac{14 \pm 2\sqrt{37}}{2} = 7 \pm \sqrt{37}$   
 $a=1 \quad b=-14 \quad c=12$   
 $\frac{(-14) \pm \sqrt{(-14)^2 - 4(1)(12)}}{2(1)}$   
 $\frac{148}{4}$   
 $37$   
 $\frac{7+\sqrt{37}}{1} = 13.08276253$   
 $\frac{7-\sqrt{37}}{1} = .9172374697$

**Practice 6:**

Together, Katy and Steve can groom all the dogs at the Barkin' Doggie Day Care in 4 hours. Alone, Katy can groom the dogs 1 hour faster than Steve can groom the dogs alone. Find the time in which each of them can groom the dogs alone.

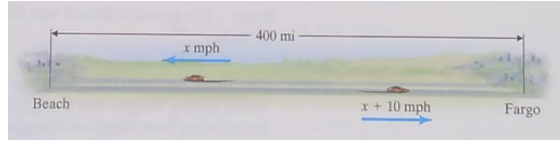
	Total Hours	Part of the Job
Katy	x - 1	$\frac{1}{x-1}$
Steve	x	$\frac{1}{x}$
Together	4	$\frac{1}{4}$

$LC: 4x(x-1)$   
 $7.5 \text{ hr}$   
 $8.5 \text{ hr}$   
 $4\left(\frac{1}{x-1} + \frac{1}{x}\right) = \frac{1}{4}$   
 $4x + 4x - 4 = x^2 - x$   
 $0 = x^2 - 9x + 4$   
 $a=1 \quad b=-9 \quad c=4$   
 $x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(4)}}{2(1)} = \frac{9 \pm \sqrt{65}}{2}$   
 $\frac{(-9)^2 - 4(1)(4)}{65}$   
 $\frac{9 + \sqrt{65}}{2} = 8.531128874$  ✓  
 $\frac{9 - \sqrt{65}}{2} = .4688711259$  ✗

**Example 7: Finding Driving Speeds**

Beach and Fargo are about 400 miles apart. A salesperson travels from Fargo to Beach one day at a certain speed. She returns to Fargo the next day and drives 10 mph faster. Her total travel time was  $14\frac{2}{3}$  hours. Find her speed to Beach and the return speed to Fargo.

**Helpful Hint**  
 Since  $d = rt$ ,  $t = \frac{d}{r}$ . The time column was completed using  $\frac{d}{r}$ .



$$14\frac{2}{3} = \frac{44}{3}$$

B

F

LCM:  $3x(x+10)$

$$\begin{array}{r} -3000 \\ +60 \\ -490 \end{array}$$

X	Y1	Plot1	Plot2	Plot3
54	-811.1			
55	-800			
56	-788.2			
57	-775			
58	-760.3			
59	-744			
60	-726			
X=60				

$$d = r \cdot t$$

$$400 \cdot x \cdot \frac{400}{x} = 50 \text{ mph}$$

$$400 \cdot (x+10) \cdot \frac{400}{x+10} = 60 \text{ mph}$$

$$\frac{3(400)(100)}{x} + \frac{100(3x)}{x+10} = \frac{44x(x+10)}{3}$$

$$300x + 3000 + 300x = 11x^2 + 110x$$

$$0 = 11x^2 - 490x - 3000$$

$$0 = 11x^2 - 550x + 60x - 3000$$

$$11x(x - 50) + 60(x - 50)$$

$$0 = (11x + 60)(x - 50)$$

$$x = 50$$

**Practice 7:**

The 36-km S-shaped Hangzhou Bay Bridge is the longest cross-sea bridge in the world, linking Ningbo and Shanghai, China. A merchant drives over the bridge one morning from Ningbo to Shanghai in very heavy traffic and returns home that night driving 50 km per hour faster. The total travel time was 1.3 hours. Find the speed to Shanghai and the return speed to Ningbo.



N

S

$$13 = \frac{13}{10}$$

$$\frac{36}{x} + \frac{36}{x+50} = \frac{13x(x+50)}{10}$$

LCM:

$$10(x)(x+50)$$

$$360(x+50) + 360x = 13x(x+50)$$

$$360x + 1800 + 360x = 13x^2 + 650x$$

$$0 = 13x^2 - 70x - 1800$$

a=13 b=-70 c=-1800

$$(-70)^2 - 4(13)(-1800) = 98500$$

$$x = \frac{-(-70) \pm \sqrt{(-70)^2 - 4(13)(-1800)}}{2(13)} = \frac{70 \pm \sqrt{98500}}{26}$$

$$\frac{(70 + \sqrt{98500})}{26} = 14.7634987$$

$$\frac{(70 - \sqrt{98500})}{26} = -9.378734482$$

$$x \approx 14.763 \approx 15 \text{ mph}$$

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1 - 17 (o), 29 - 35 (o), 45 - 71 (o)