11.5 Volume of Prisms and Cylinders

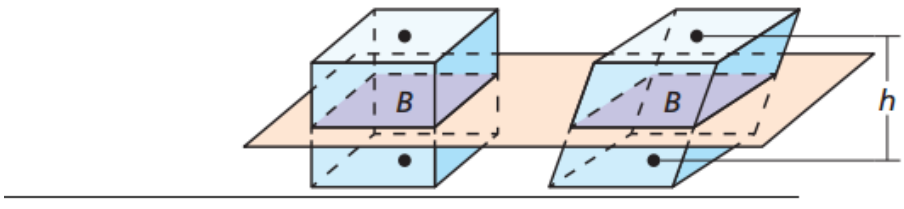
How much coffee can go in the coffee mug? Discuss how we would figure this information out.

How much does Godzilla weigh? Discuss as table before moving picture to find out.

Mar 8-9:56 AM

Finding Volumes of Prisms and Cylinders

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3). **Cavalieri's Principle**, named after Bonaventura Cavalieri (1598–1647), states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. The prisms below have equal heights h and equal cross-sectional areas B at every level. By Cavalieri's Principle, the prisms have the same volume.



May 4-1:27 PM

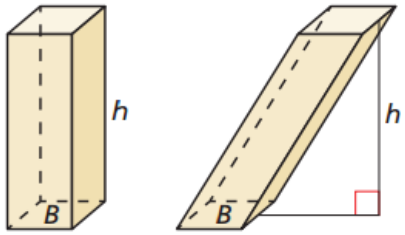
Core Concept

Volume of a Prism

The volume V of a prism is

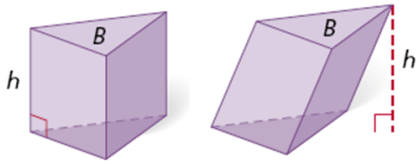
$$V = Bh$$

where B is the area of a base and h is the height.

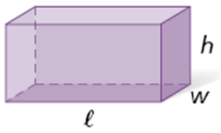


Volume of a Prism

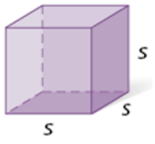
The volume of a prism with base area B and height h is $V = Bh$.



The volume of a right rectangular prism with length ℓ, width w, and height h is $V = \ell wh$.



The volume of a cube with edge length s is $V = s^3$.



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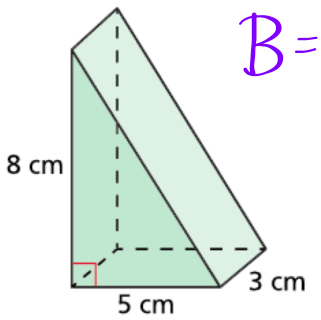
Example 1:

a.



$B = \frac{1}{2}bh$

$V = Bh$

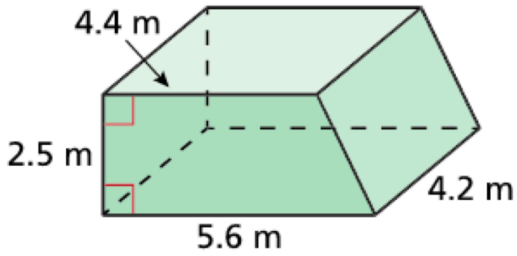
$= \left(\frac{1}{2}bh\right)h$

$= \frac{1}{2}(5)(8)(3)$

$= 60\text{cm}^3$

(1/2)(5.6+4.4)(2.5)
12.5
Ans*4.2
52.5

b.



$V = Bh$

$B = \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(5.6 + 4.4)(2.5)$

$= 12.5$

$V = (12.5)(4.2)$

$V = 52.5\text{m}^3$

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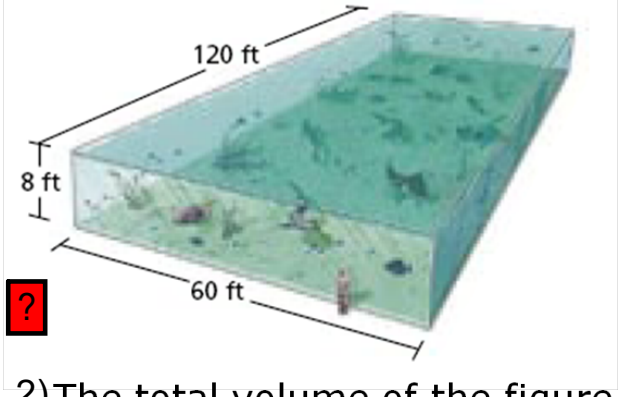
Practice:

1) If the volume of an aquarium is 57600 ft^3, find it's height.



$V = Bh = lwh$

$57{,}600 = (20)(60)(h)$

$57{,}600 = 7200h$

$h = 8$ ft

2) The total volume of the figure is the sum of the volumes.

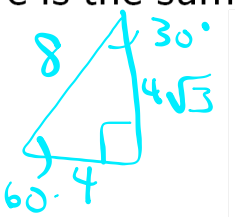
$V = \triangle + \square$

$= (\frac{1}{2}bh)h + lwh$

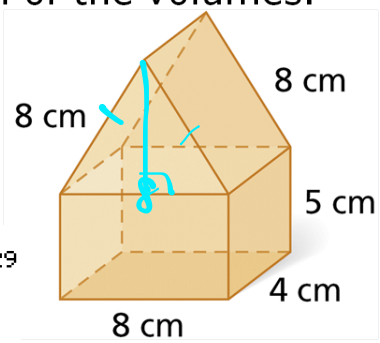
$= \frac{1}{2}(8)(4\sqrt{3})(4) + (8)(4)(5)$

$= 64\sqrt{3} + 160$ cm^3

≈ 270.851 cm^3



16√3+160
187.7128129



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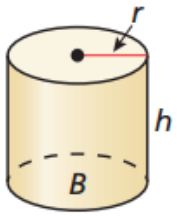
Core Concept

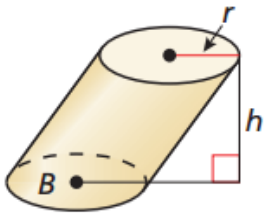
Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of a base.





REAL-LIFE EXAMPLES

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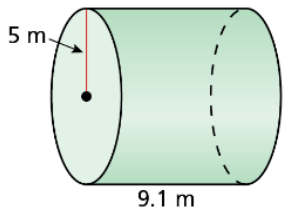
Example 2:

Find the volume of each cylinder.

a.

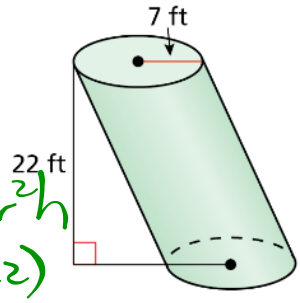


$V = Bh = \pi r^2 h$

$= \pi(5)^2(9.1)$

227.5π, or about 714.71 cubic meters

b.

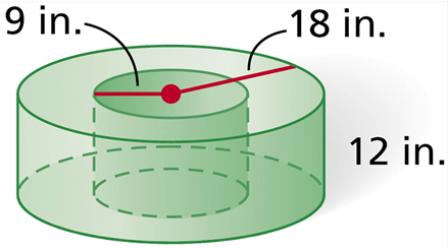


$V = Bh = \pi r^2 h$

$= \pi(7)^2(22)$

1078π, or about 3386.64 cubic feet

c. Find the volume of the composite figure.



$V =$ ▭ − ○

$= \pi r^2 h - \pi r^2 h$

$= \pi(18)^2(12) - \pi(9)^2(12)$

$= 2916\pi \text{ in}^3$

$\approx 9160.9 \text{ in}^3$

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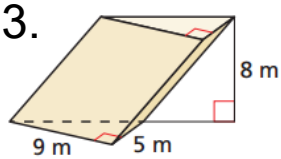
Practice:

Find the volume of the solid.

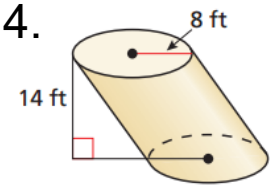
3.



$V = Bh = \frac{1}{2}bhh$

$= \frac{1}{2}(5)(9)(8) = 180 \text{ m}^3$

4.



$V = Bh =$

$= \pi r^2 h$

$= \pi(8)^2(14)$

$= 896\pi \text{ ft}^3$

5. One cup is equal to 14.4375 in^3. If 1 cylinder measuring cup has a radius of 2 in., what is the height? If the radius is 1.5 in., what is the height?

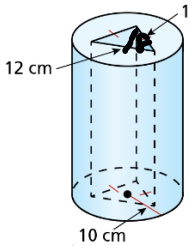
$14.4375 = \pi(2)^2 h$

$1.149 \text{ in} = h$

$14.4375 = \pi(1.5)^2 h$

$2.043 \text{ in} = h$

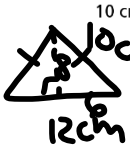
6. Find the volume of the composite solid.



$V =$ ▭ − △

$V = \pi r^2 h - (\frac{1}{2}bh)h$

$= \pi(10)^2(32) - \frac{1}{2}(12)(8)(32)$



$= 3200\pi - 1536 \text{ cm}^3$

$\approx 8517.1 \text{ cm}^3$

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11.5 Volumes of prisms and cylinders with work

Using the Formula for Density

Density is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

Example 3:

In figure 1, a cylinder with a diameter of 12 cm is filled with water to a height of 8 cm. In figure 2 a rock is submerged in the cylinder. Find the approximate volume of the rock.

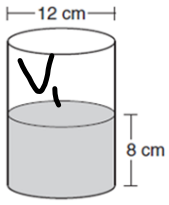

Figure 1

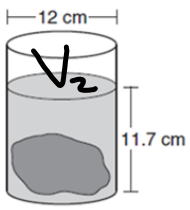

Figure 2

$V_R = V_2 - V_1$

$= 421.2\pi - 288\pi$

$= 133.2\pi \text{ cm}^3$

$\approx 418.46 \text{ cm}^3$

$V_1 = Bh = \pi r^2 h$

$= \pi(6)^2(8)$

$= 288\pi \text{ cm}^3$

$V_2 = \pi(6)^2(11.7)$

$= 421.2\pi \text{ cm}^3$

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Practice 6: The density of water is 1000 kilograms per cubic meter. Find the mass of 1 cubic foot of water. Use the fact that 1 foot = 0.3048 meters.

$\text{Density} = \frac{m}{V}$

$1000 = \frac{m}{0.3048}$

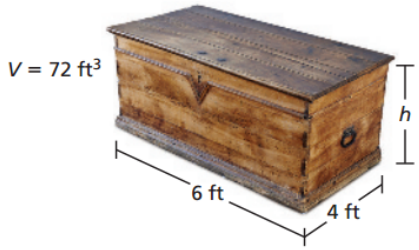
$m = 304.8 \text{ kg}$

Practice 7: You are building a rectangular chest. You want the length to be 6 feet, the width to be 4 feet, and the volume to be 72 cubic feet. What should the height be?

$V = lwh$

$72 = (6)(4)h$

$3 \text{ ft} = h$



Practice 8: You are building a 3-foot tall dresser. You want the volume to be 42 cubic feet. What should the area of the base be? Give a possible length and width.

$V = lwh = Bh$

$42 = B(3)$

$B = 14 \text{ ft}$

$l = 7 \text{ ft}$

$w = 2 \text{ ft}$

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HW: pg. 631

A: 15, 23, 27, 29, 33, 35, 37, 41, 45, 47, 53, 55 - 57

B: 5, 9, 11, 13, 15, 21, 23, 25, 27, 29, 33, 35, 37, 55 - 57

C: 1 - 37 (o), 55 - 57

ANSWERS:

56. 406.3 cm^2

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