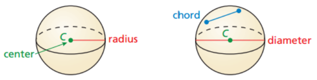
11.8 Surface Area and volume of sphere with work

11.8 Surface Area and Volume of a Sphere

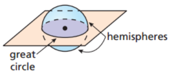
Finding Surface Areas of Spheres

A *sphere* is the set of all points in space equidistant from a given point. This point is called the *center* of the sphere. A *radius* of a sphere is a segment from the center to a point on the sphere. A **chord of a sphere** is a segment whose endpoints are on the sphere. A *diameter* of a sphere is a chord that contains the center.



As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

If a plane intersects a sphere, then the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called *hemispheres*.



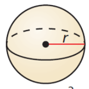
Core Concept

Surface Area of a Sphere

The surface area S of a sphere is

$S = 4\pi r^2$

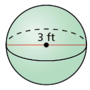
where r is the radius of the sphere.



$S = 4\pi r^2$

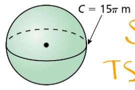
Example 1: Find the surface area of each sphere.

a.



$T = 4\pi r^2$
$= 4\pi(1.5)^2$
$= \boxed{9\pi\ ft^2}$

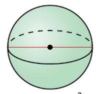
b.



$S = 4\pi r^2$
$TSA = 4\pi\left(\frac{15}{2}\right)^2$
$= 4\pi\left(\frac{225}{4}\right)$
$= \boxed{225\pi\ m^2}$

$C = 2\pi r$
$15\cancel{\pi} = 2\cancel{\pi} r$
$\frac{15}{2} = r$

Example 2: Find the diameter of the sphere.


$S = 144\pi\ cm^2$

$144\cancel{\pi} = 4\cancel{\pi} r^2$
$36 = r^2$
$\pm 6 = r$
$r = 6cm$
$\boxed{d = 12cm}$

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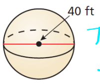
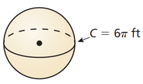
Practice: Find the surface area of the sphere.

1.



$T = 4\pi r^2$
$= 4\pi(20)^2$
$= \boxed{1600\pi\ ft^2}$

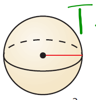
2.



$C = 2\pi r$
$6\cancel{\pi} = 2\cancel{\pi} r$
$3 = r$

$T = 4\pi r^2$
$= 4(\pi)(3)^2$
$= \boxed{36\pi\ ft^2}$

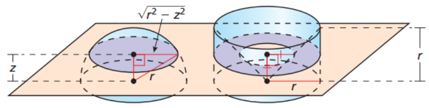
3. Find the radius of the sphere.


$S = 30\pi\ m^2$

$T = 4\pi r^2$
$30\cancel{\pi} = 4\cancel{\pi} r^2$
$\sqrt{\frac{15}{2}} = \sqrt{r^2}$
$\boxed{r = \sqrt{\frac{15}{2}}\ m}$

Finding Volumes of Spheres

The figure shows a hemisphere and a cylinder with a cone removed. A plane parallel to their bases intersects the solids z units above their bases.



Using the AA Similarity Theorem (Theorem 8.3), you can show that the radius of the cross section of the cone at height z is z. The area of the cross section formed by the plane is $\pi(r^2 - z^2)$ for both solids. Because the solids have the same height and the same cross-sectional area at every level, they have the same volume by Cavalieri's Principle.

$V_{\text{hemisphere}} = V_{\text{cylinder}} - V_{\text{cone}}$
$= \pi r^2(r) - \frac{1}{3}\pi r^2(r)$
$= \frac{2}{3}\pi r^3$

So, the volume of a sphere of radius r is

$2 \cdot V_{\text{hemisphere}} = 2 \cdot \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3.$

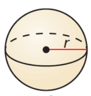
Core Concept

Volume of a Sphere

The volume V of a sphere is

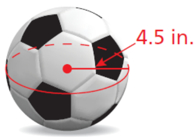
$V = \frac{4}{3}\pi r^3$

where r is the radius of the sphere.



$V = \frac{4}{3}\pi r^3$

Example 3: Find the volume of the soccer ball.

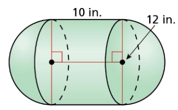


$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(4.5)^3$
$= \boxed{121.5\pi\ in^3}$

Example 4: The surface area of a sphere is 676π square inches. Find the volume of the sphere.

$T = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$
$676\cancel{\pi} = 4\cancel{\pi} r^2$
$169 = r^2$
$\boxed{13^{in} = r}$
$V = \frac{4}{3}\pi(13)^3$
$\boxed{= 2929.333\pi\ in^3}$

Practice 4: Find the volume of the composite solid.



The volume is 648π, or about 2036 cubic inches.

$V = ◯ + ⌸$
$= \frac{4}{3}\pi r^3 + \pi r^2 h$
$= \frac{4}{3}\pi(6)^3 + \pi(6)^2(10)$

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11.8 Surface Area and volume of sphere with work

HW: pg. 652

A: 21, 23, 29, 33, 35, 39, 41, 47 - 51

B: 1, 5, 9, 11, 17, 19, 21, 25, 31, 35, 39, 48 - 51

C: 1 - 35 (o), 48 - 51

ANSWERS:

48. a) $\approx 54.45 \text{ ft}^3$, b) $\approx 28{,}500.53 \text{ cm}^3$, c) $\approx 2077.64 \text{ m}^3$, d) $\approx 522{,}170.40 \text{ in}^3$

50. SA $\approx 113.10 \text{ in}^2$ V $\approx 75.40 \text{ in}^3$

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