Good Luck to Key Date _____ Pd ____

ACT Mid-TEST: 11th Graders

Since there are 15 questions you get 20 minutes. Do your best! Place the CAPITAL LETTER in the box provided.

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MATHEMATICS TEST

60 Minutes—60 Questions

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates arithmetic mean.

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1. An artist makes a profit of $(500p - p^2)$ dollars from selling p paintings. What is the fewest number of paintings the artist can sell to make a profit of at least $60,000?

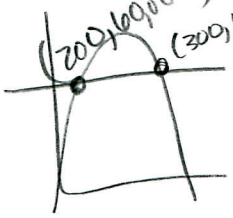
C

A. 100
B. 150
C. 200
D. 300
E. 600

$500p - p^2 > 60000$

$p(500 - p) > 60000$

(200, 60000) (300, 60000)

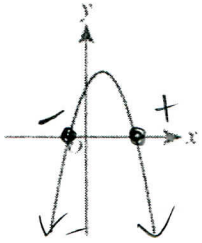


Window
0
1000
100
0
65000
10000

2. The ~~equation~~ $y = ax^2 + bx + c$ is graphed in the standard (x, y) coordinate plane below for real values of a, b, and c. When y = 0, which of the following best describes the solutions for x?

H

F. 2 distinct positive real solutions
G. 2 distinct ~~negative real~~ solutions
H. 1 positive real solution and 1 negative real solutions
J. 2 real solutions that are not distinct
K. 2 distinct solutions that are not real



3. What is the product of the complex numbers (-3*i* + 4) and (3*i* + 4)?

C

A. 1
B. 7
C. 25
D. -7 + 24*i*
E. 7 + 24*i*

$(-3i+4)(3i+4)$

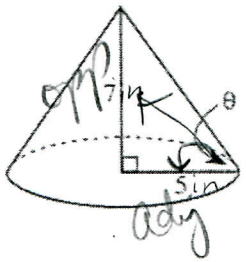
$-9i^2 - 12i + 12i + 16$

$9 + 16 = 25$

4. The radius of the base of the right circular cone shown below is 5 inches, and the height of the cone is 7 inches. Solving which of the following equations gives the measure, Θ, of the anlge formed by a slant height of the cone and a radius?

G

F. $tan\ \theta = \frac{5}{7}$
G. $tan\ \theta = \frac{7}{5}$
H. $sin\ \theta = \frac{5}{7}$
J. $sin\ \theta = \frac{7}{5}$
K. $cos\ \theta = \frac{7}{5}$



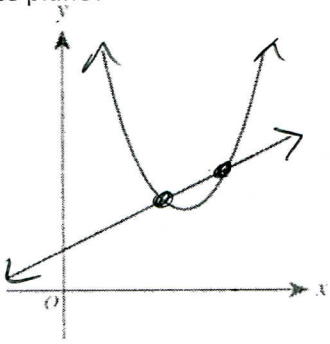
adj

$\tan \frac{7}{5}$

5. Which of the following describes a true relationship between the functions $f(x) = (x-3)^2 + 2$ and $g(x) = \frac{1}{2}x + 1$ graphed below in the standard (x, y) coordinate plane?

A

A. $f(x) = g(x)$ for exactly 2 values of x
B. $f(x) = g(x)$ for exactly 1 value of x
C. $f(x) < g(x)$ for all x
D. $f(x) > g(x)$ for all x
E. $f(x)$ is the inverse of $g(x)$



6. Given $f(x) = x - \frac{1}{x}$ and $g(x) = \frac{1}{x}$, what is $f\left(g\left(\frac{1}{2}\right)\right)$?

K

F. -3
G. $-\frac{3}{2}$
H. $-\frac{2}{3}$
J. 0
K. $\frac{3}{2}$

$g(\frac{1}{2}) = \frac{1}{\frac{1}{2}} = 2$

$f(2) = 2 - \frac{1}{2} = 1.5$

7. A formula to estimate the monthly payment, p dollars, on a short-term loan is $p = \frac{\frac{1}{2}ary + a}{12y}$

Where a dollars is the amount of the loan, r is the annual interest rate expressed as s decimal, and y years is the length of the loan. When a is multiplied by 2, what is the effect on p?

D

A. p is divided by 6
B. p is divided by 2
C. p does not change
D. p is multiplied by 2
E. p is multiplied by 4

8. $\frac{4}{\sqrt{2}} + \frac{2}{\sqrt{3}} = ?$

$\frac{4\sqrt{3}}{\sqrt{2}\sqrt{3}} + \frac{2\sqrt{2}}{\sqrt{3}\sqrt{2}} = \frac{4\sqrt{3}}{\sqrt{6}} + \frac{2\sqrt{2}}{\sqrt{6}} = \frac{4\sqrt{3}+2\sqrt{2}}{\sqrt{6}}$

G

F. $\frac{4\sqrt{3}+2\sqrt{2}}{\sqrt{5}}$
G. $\frac{4\sqrt{3}+2\sqrt{2}}{\sqrt{6}}$
H. $\frac{6}{\sqrt{2}+\sqrt{3}}$
J. $\frac{6}{\sqrt{5}}$
K. $\frac{8}{\sqrt{6}}$

9. Given that $a\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} x & 27 \\ y & z \end{bmatrix}$ for some real number a, what is x + z?

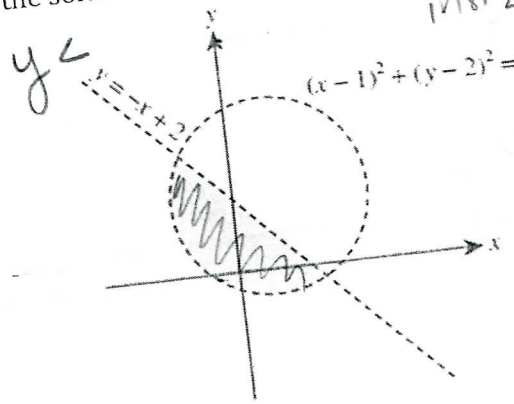
$6a = 27$
$a = \frac{27}{6}$
$x = 2a$
$z = 4a$
$2a + 4a \Rightarrow 2\left(\frac{27}{6}\right) + 4\left(\frac{27}{6}\right) = 27$

D

A. $\frac{4}{3}$
B. $\frac{27}{2}$
C. 26
D. 27
E. 48

10. The shaded region in the graph below represents the solution set to which of the following systems of inequalities?

inside 9



F

F. $\begin{cases} y < -x + 2 \\ (x-1)^2 + (y-2)^2 < 9 \end{cases}$
G. $\begin{cases} y \leq -x + 2 \\ (x-1)^2 + (y-2)^2 < 9 \end{cases}$
H. $\begin{cases} y > -x + 2 \\ (x-1)^2 + (y-2)^2 > 9 \end{cases}$
J. $\begin{cases} y < -x + 2 \\ (x-1)^2 + (y-2)^2 > 9 \end{cases}$
K. $\begin{cases} (y-2) < 3 \\ (x-1) > 3 \end{cases}$

2 lines

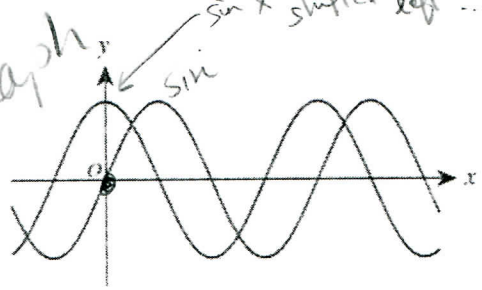
11. The functions $y = \sin x$ and $y = \sin(x + a) + b$, for constants a and b, are graphed in the standard (x, y) coordinate plane below. The functions have the same maximum value. One of the following statements about the values of a and b is true. Which statement is it?

pre calc transformation

A

A. $x < 0$ and $b = 0$
B. $a < 0$ and $b > 0$
C. $a = 0$ and $b > 0$
D. $a > 0$ and $b < 0$
E. $a > 0$ and $b > 0$

plug in and graph



sin x shifted left ... so a is negative

b=0 since no vertical shift

12. Which of the following number line graphs shows the solution set to the inequality $|x - 5| < -1$?

K

F.


G.


H.


J.


K.

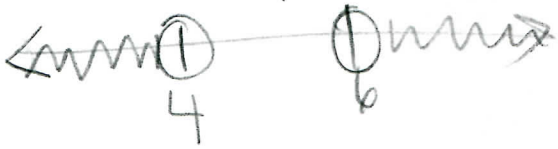

(empty set)

$x - 5 < -1$
$+5 \quad +5$
$x < 4$

$x - 5 > 1$
$+5 \quad +5$
$x > 6$

or

never overlap



13. A copy machine makes 60 copies per minute. A second copy machine makes 80 copies per minute. The second machine starts making copies 2 minutes after the first machine starts. Both machines stop making copies 8 minutes after the first machine started. Together the 2 machines make how many copies?

E

A. 480
B. 600
C. 680
D. 720
E. 960

$\frac{60 \text{ copies}}{1 \text{ min}}$ $\frac{80 \text{ copies}}{1 \text{ min}} = \frac{7}{240}$ together

$(60)\,2 \text{ mins} = 120$

$120 + 840$

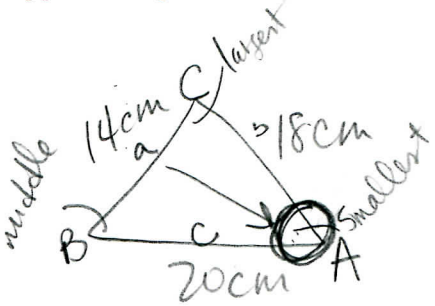
$(60)6 + (80)(6) = 840$

960

geo

14. The sides of an acute triangle measure 14 cm, 18 cm, and 20 cm, respectively. Which of the following equations, when solved for θ, gives the measure of the smallest angle of the triangle?

(Note: For any triangle with sides of length a, b, and c that are opposite angles A, B, and C, respectively, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ and $c^2 = a^2 + b^2 - 2ab\cos C$.)



J

F. $\frac{\sin\theta}{14} = \frac{1}{18}$ $18\sin\theta = 14 \Rightarrow \sin^{-1}\left(\frac{14}{18}\right) = 51°$

G. $\frac{\sin\theta}{14} = \frac{1}{20}$ $20\sin\theta = 14 \Rightarrow \sin^{-1}\left(\frac{14}{20}\right) = 44°$

H. $\frac{\sin\theta}{20} = \frac{1}{14}$ $14\sin\theta = 20 \Rightarrow \sin^{-1}\left(\frac{20}{14}\right) = $ NP

J. $14^2 = (18^2 + 20^2) - 2(18)(20)\cos\theta$ $\frac{14^2 - 18^2 - 20^2}{-2(18)(20)} = \cos\theta$

$\cos^{-1}\left(\frac{14^2 - 18^2 - 20^2}{-2(18)(20)}\right) = 43°$

K. $20^2 = 14^2 + 18^2 - 2(14)(18)\cos\theta$

$\cos^{-1}\left(\frac{20^2 - 14^2 - 18^2}{-2(14)(18)}\right) = 76°$

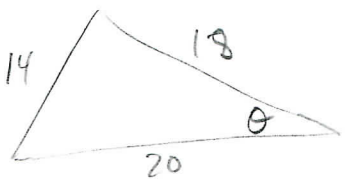
15. Discount tickets to a basketball tournament sell for $4.00 each. Enrico spent $60.00 on discount tickets, $37.50 less than if he had bought the tickets at the regular price. What was the regular ticket price?

C

A. $2.50
B. $6.40
C. $6.50
D. $7.50
E. $11.00

$\frac{60}{4} = 15$

$\frac{\$97.50}{15} = 6.50$



$14^2 = 18^2 + 20^2 - 2(18)(20)\cos\theta$