2.8 Solving Linear Inequalities

OBJECTIVE 1: Graphing Solution Sets to Linear Inequalities & Using Interval Notation

< means "is less than"

> means "is greater than"

$\leq$ means "is less than or equal to"

$\geq$ means "is greater than or equal to"

Equations	Inequalities
x = 3	x $\leq$ 3
5n - 6 = 14	5n - 6 < 14
12 = 7 - 3y	12 $\geq$ 7 - 3y
(x/4) - 6 = 1	(x/4) - 6 > 1

Linear Inequality in One Variable

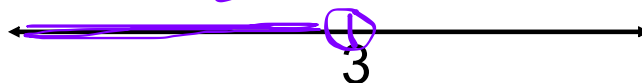A **linear inequality in one variable** is an inequality that can be written in the form

$$ax + b < c$$

where a, b, and c are real numbers and a is not 0.

A solution of an inequality is a value of the variable that makes the inequality a true statement.

A solution set is a set of all solutions.

So... if x < 3 then... {x|x<3} LOL

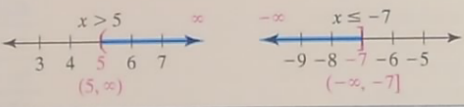$\bigcirc$ < >

$\leq$ $\geq$

3

Set builder notation was used on the last slide to show x < 3, but we can also use interval notation.

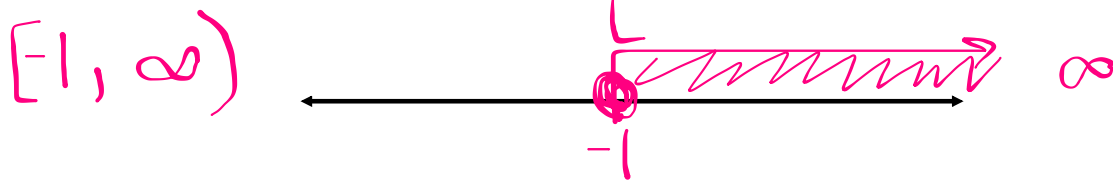| Set Builder	Inequality	Interval Notation
$\{x \mid x < 3\}$	$x < 3$	$(-\infty, 3)$

○ < > ( )   ● ≤ ≥ [ ]

> **Helpful Hint**
>
> When writing an inequality in interval notation, it may be easier to graph the inequality first, then write it in interval notation. To help, think of the number line as approaching $-\infty$ to the left and $+\infty$ or ∞ to the right. Then simply write the interval notation by following your shading from left to right.
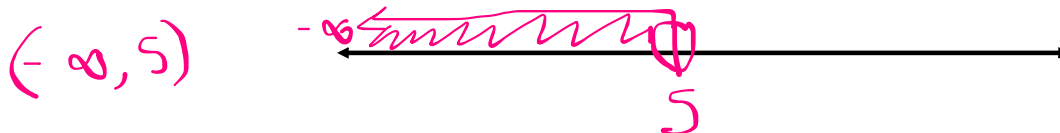>
> $x > 5$ ∞ $-\infty$ $x \leq -7$
>
> 3 4 5 6 7 -9 -8 -7 -6 -5
>
> $(5, \infty)$ $(-\infty, -7]$

Example 1: Graph $x \geq -1$.
Then write the solutions in interval notation.

LoL

$[-1, \infty)$

-1 ∞

Practice 1: Graph $x < 5$.
Then write the solutions in interval notation.

$(-\infty, 5)$

$-\infty$ 5

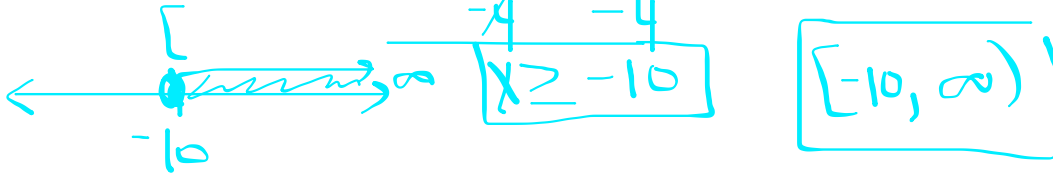**2**

OBJECTIVE 2: Solving Linear Inequalities

Addition Property of Inequality

If a, b, and c are real numbers, then

$$a < b \quad \text{and} \quad a + c < b + c$$

are equivalent inequalities.

Example 2: Solve x + 4 ≥ - 6

$$-4 \quad -4$$
$$x \geq -10$$
$$[-10, \infty)$$

Practice 2: Solve x + 11 ≥ 6

$$-11 \quad -11$$
$$x \geq -5$$
$$[-5, \infty)$$
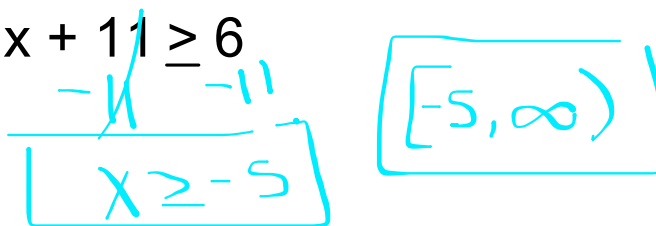
Helpful Hint

Notice that any number less than or equal to -10 is a solution of $x \leq -10$. For example, solutions include

$$-10, \ -200, \ -11\frac{1}{2}, \ -7\pi, \ -\sqrt{130}, \ -50.3$$

Multiplication Property of Inequality

1. If a, b, and c are real numbers, and c is positive, then

$$a < b \quad \text{and} \quad ac < bc$$

are equivalent inequalities.

2. If a, b, and c are real numbers, and c is negative, then

$$a < b \quad \text{and} \quad ac > bc$$
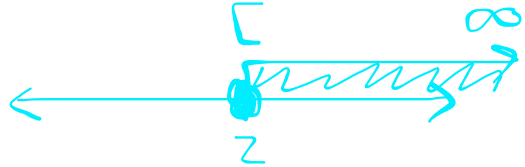
are equivalent inequalities.

Helpful Hint

Whenever both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality symbol must be reversed to form an equivalent inequality.

Example 3: Solve $-2x \leq -4$. Graph the solution set and write it in interval notation.

$$\frac{-2x \leq -4}{-2} \quad \frac{}{-2}$$

$$x \geq 2$$

Helpful Hint

Don't forget to reverse the direction of the inequality symbol.

$$[2, \infty)$$

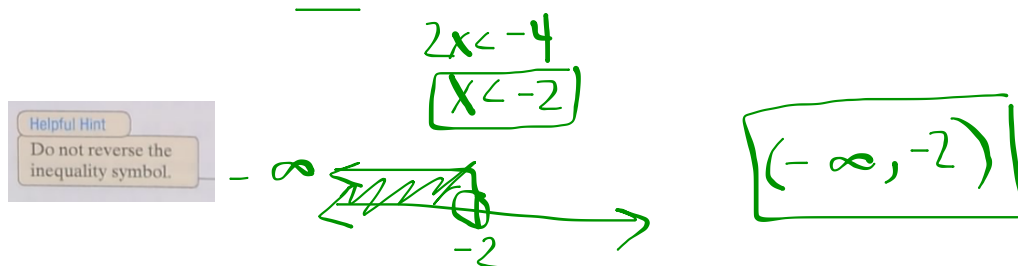**Practice 3:** Solve $-5x \geq -15$. Graph the solution set and write it in interval notation.

$$\frac{-5x \geq -15}{-5} \quad \frac{}{-5}$$

$$x \leq 3$$

$$(-\infty, 3]$$

Example 4: Solve 2x < - 4. Graph the solution set and write it in interval notation.

$$2x < -4$$

$$\boxed{x < -2}$$

Helpful Hint
Do not reverse the inequality symbol.

$$\boxed{(-\infty, -2)}$$

-2

Practice 4: Solve 3x > - 9. Graph the solution set and write it in interval notation.

$$3x > -9$$

$$\boxed{x > -3}$$

$$\boxed{(-3, \infty)}$$

-3
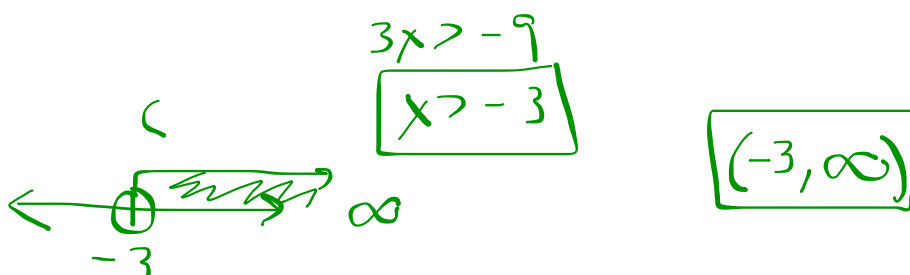
✔ CONCEPT CHECK
Fill in the blank with <, >, ≤, or ≥.

a. Since $-8 < -4$, then $3(-8)\underline{\phantom{<}<}3(-4)$.

b. Since $5 \geq -2$, then $\dfrac{5}{-7}\underline{\leq}\dfrac{-2}{-7}$.

c. If $a < b$, then $2a\underline{\phantom{<}<}2b$.

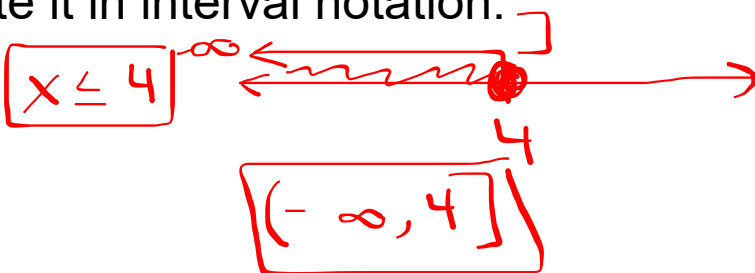d. If $a \geq b$, then $\dfrac{a}{-3}\underline{\leq}\dfrac{b}{-3}$.

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Solving Linear Inequalities in One Variable

Step 1. Clear the inequality of fractions by multiplying both sides of the inequality by the least common denominator (LCD) of all fractions in the inequality.

Step 2. Remove grouping symbols such as parentheses by using the distributive property.

Step 3. Simplify each side of the inequality by combining like terms.

Step 4. Write the inequality with variable terms on one side and numbers on the other side by using the addition property of inequality.

Step 5. Get the variable alone by using the multiplication property of inequality.

Helpful Hint

Don't forget that if both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality symbol must be reversed.
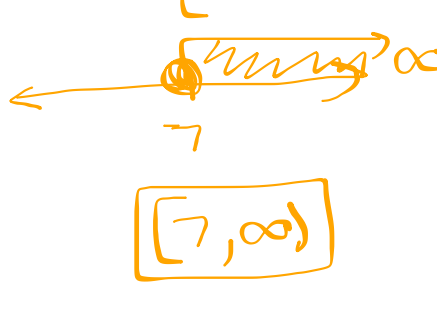
Example 5: Solve -4x + 7 ≥ - 9. Graph the solution set and write it in interval notation.

$$-4x + 7 \geq -9$$
$$ -7 \quad -7$$
$$\frac{-4x}{-4} \geq \frac{-16}{-4}$$

$$x \leq 4$$

$$(-\infty, 4]$$

Practice 5: Solve 45 - 7x ≤ - 4. Graph the solution set and write it in interval notation.

$$45 - 7x \leq -4$$
$$-45 \qquad -45$$
$$\frac{-7x}{-7} \geq \frac{-49}{-7}$$

$$x \geq 7$$

$$[7, \infty)$$

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Example 6: Solve $2x + 7 \leq x - 11$. Graph the solution set and write it in interval notation.

$$2x + 7 \leq x - 11$$
$$\underline{-x \qquad -x}$$
$$x + 7 \leq -11$$
$$\underline{-7 \qquad -7}$$
$$\boxed{x \leq -18}$$

$$-\infty \quad \overleftarrow{} \bullet \longrightarrow$$
$$-18$$

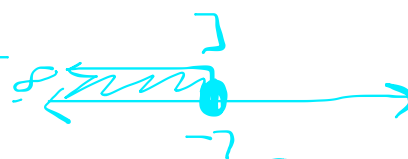$$\boxed{(-\infty, -18\,]}$$

Practice 6: Solve $3x + 20 \leq 2x + 13$. Graph the solution set and write it in interval notation.

$$3x + 20 \leq 2x + 13$$
$$\underline{-2x \qquad -2x}$$
$$x + 20 \leq 13$$
$$\underline{-20 \qquad -20}$$
$$\boxed{x \leq -7}$$

$$\overleftarrow{} \bullet \longrightarrow$$
$$-7$$

$$\boxed{(-\infty, 7]}$$
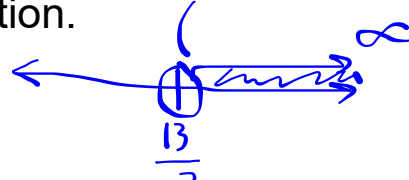
Example 7: Solve $-5x + 7 < 2(x - 3)$. Graph the solution set and write it in interval notation.

$$\boxed{x > \frac{13}{7}}$$

$$-5x + 7 < 2x - 6$$
$$\underline{+5x \qquad +5x}$$
$$7 < 7x - 6$$
$$\underline{+6 \qquad +6}$$
$$\frac{13}{7} < \frac{7x}{7}$$

$$\longleftarrow \circ \overrightarrow{} \infty$$
$$\frac{13}{7}$$

$$\boxed{\left(\frac{13}{7}, \infty\right)}$$

Practice 7: Solve $6 - 5x > 3(x - 4)$. Graph the solution set and write it in interval notation.
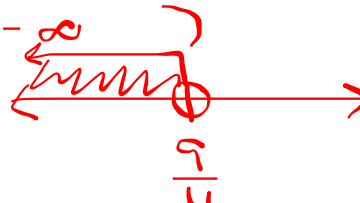
$$\boxed{x < \frac{9}{4}}$$

$$6 - 5x > 3x - 12$$
$$\underline{+5x \qquad +5x}$$
$$6 > 8x - 12$$
$$\underline{+12 \qquad +12}$$
$$\frac{9}{4} = 2 \div \frac{18}{8} > \frac{8x}{8}$$

$$-\infty \quad \overleftarrow{} \circ \longrightarrow$$
$$\frac{9}{4}$$

$$\boxed{\left(-\infty, \frac{9}{4}\right)}$$

Example 8: Solve $2(x - 3) - 5 \leq 3(x + 2) - 18$. Graph the solution set and write it in interval notation.

$2x - 6 - 5 \leq 3x + 6 - 18$

$2x - 11 < 3x - 12$

$-2x \quad -2x$

$-11 < x - 12$

$+12 \quad +12$

$1 < x$

$x > 1$

$(1, \infty)$

Practice 8: Solve $3(x - 4) - 5 \leq 5(x - 1) - 12$. Graph the solution set and write it in interval notation.

$3x - 12 - 5 \leq 5x - 5 - 12$

$3x - 17 \leq 5x - 17$

$3x \leq 5x$

$-3x \quad -3x$

$\dfrac{0}{2} \leq \dfrac{2x}{2}$

$0 \leq x$

$\dfrac{0}{K} \quad \dfrac{N}{U}$

$x \geq 0$

$[0, \infty)$

HW DAY ONE

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1 - 27 (o)