3.2 Measures of Variation with work

1. Median, Mode, mean

3.1 Answers

5. $\overline{x} = 5$; median = 6, mode = 2
7. $\overline{x} = 5$, median = 5.5, mode = 2
9. a) No, the sum of the data does not change.
   b) No, changing extreme data values does not affect the median.
   c) Yes, depending on which data value occurs most frequently after the data are changed.
11. Mean, median, and mode are approximately equal.
15. The supervisor has a legitimate concern because at least half the clients rated the employee below satisfactory. From the information given, it seems that this employee is very inconsistent in her performance.
17. a) Mode = 2, median = 3, mean = 4.6
    b) Mode = 10, median = 8, mean = 9.6
    c) Corresponding values are 5 times the original averages. In general, multiplying each data value by a constant c results in the mode, median, and mean changing by a factor of c.
19. Mean ≈ 167.3°F; median = 171°F; mode = 178°F

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3.2 Measures of Variation

Essential Question:

What does variation mean to statistics and how is it useful?

Why is the coefficient of variation important?

Focus Points:

- Find the range, variance, and standard deviation.
- Compute the coefficient of variation from raw data.

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The range is a measure of variation. The range is found by taking the difference between the largest and smallest values of a data distribution.

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Example 1: Bakery Range

A large bakery regularly orders cartons of Maine blueberries. The average weight of the cartons is supposed to be 22 ounces. Random samples of cartons from a supplier were weighed. The weights in ounces of the cartons were:

17 22 22 22 27

a) Compute the range of carton weights. $27 - 17 = \boxed{10}$

b) Compute the mean weight of cartons from the supplier.

$\overline{x} = \frac{17+22+22+22+27}{5} = \boxed{22}$ $\tilde{x} = 22$

Mode = 22

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Variance and Standard Deviation

There are many ways to measure data spread, and the sample standard deviation *s* is only one way (range is another). However, just as standard time is the time to which most people refer, standard deviation is the measure of data spread to which most people refer.

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HOW TO COMPUTE THE SAMPLE VARIANCE AND SAMPLE STANDARD DEVIATION

Quantity	Description
x	data value, outcome
$Mean = \overline{x} = \frac{\sum x}{n}$	average of the data values, what you "expect"
$x - \overline{x}$	difference between what happened and what was expected
$\sum (x - \overline{x})^2$	Sum of Squares
$s^2 = \frac{\sum (x-\overline{x})^2}{n-1}$	Sample Variance is s^2
$s = \sqrt{\frac{\sum (x-\overline{x})^2}{n-1}}$	Sample Standard Deviation, s

S = sample   σ = pop.

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3.2 Measures of Variation with work

Example 2: Greenhouse Standard Deviation

Lady Landon's Greenhouse was commissioned to develop an extra large rose for the Rose Bowl Parade. A SRS of blossoms from Hybrid A bushes yielded the following diameters (in inches) for mature peak blooms.

2 3 3 8 10 10

Find the sample variance and standard deviation.

$\bar{x} = \frac{2+3+3+8+10+10}{6} = 6$

$\sigma^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

$= \frac{\Sigma[(2-6)+(3-6)+(3-6)+(8-6)+(10-6)+(10-6)]^2}{5}$

$\sigma = \sqrt{\sigma^2}$

1-Var Stats
$\bar{x}$=6
Σx=36
Σx²=286
Sx=3.741657387
σx=3.415650255
↓n=6

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Practice: Greenhouse Standard Deviation

Crazy Correll's Greenhouse was commissioned to develop an extra large rose for the Thanksgiving Day Parade. A SRS of blossoms from Hybrid B bushes yielded the following widths (in inches) for mature peak blooms.

5 5 5 6 7 8

Find the sample variance and standard deviation.

$\bar{x} = \frac{5+5+5+6+7+8}{6} = \frac{36}{6} = \boxed{6}$

1-Var Stats
$\bar{x}$=6
Σx=36
Σx²=224
Sx=1.264911064
σx=1.154700538
↓n=6

$\boxed{S_x = 1.265}$

$S_x^2 = (1.265)^2 = \boxed{1.6}$

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ROUNDING ERRORS

Do not round until your final answer or you will end with a huge rounding error. To minimize rounding errors, leave exact until your final answer, and then round to 3 decimal places.

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WHAT DO MEASURES OF VARIATION TELL US?

- The range tells us the difference between the highest and lowest values, it tells us about the spread of the data, but not how close the data is to the mean.
- The sample standard deviation is based on the difference between *each* data value and the mean of the data set. The standard deviation gives an average of data spread around the mean. The larger the standard deviation the more spread out the data are around the mean. A smaller standard deviation indicates that the data tend to be closer to the mean.
- The variance tells us the square of the standard deviation. It also is a measure of data spread around the mean.

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POPULATION PARAMETERS

Population mean = μ = $\frac{\sum x}{N}$

Population variance = $\sigma^2 = \frac{\sum (x-\mu)^2}{N}$

Population standard deviation = σ = $\sqrt{\frac{\sum (x-\mu)^2}{N}}$

where N = the number of data values in the population and x represents the individual data values of the population.

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	Sample	Population
Mean	$\bar{x}$	μ
Standard Deviation	s	σ
Variance	s^2	σ^2
Data Set	n	N

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3.2 Measures of Variation with work

Stat, CALC, 1-Var Stats

(after data is typed into L_1)

```
NORMAL FLOAT AUTO REAL RADIAN MP
1-Var Stats
x̄=35.95
Σx=719
Σx²=32089
Sx=18.12377038
σx=17.66486626
n=20
minX=14
↓Q1=21.5
```

$S_x^2 = (18.124)^2$

$\sigma_x^2 = (17.665)^2$

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COEFFICIENT OF VARIATION

Standard deviation is not easily compared between populations because of different units of measure. Instead, statisticians use coefficient of variation, which is percent to compare populations and data.

If $\bar{x}$ and s represent the sample mean and sample standard deviation, respectively then the sample coefficient of variation (CV) is defined as

CV = $\frac{s}{\bar{x}} \cdot 100\%$

Similarly, if μ and σ represent the population then their formula is

CV = $\frac{\sigma}{\mu} \cdot 100\%$

$CV = \_\_\_ \%$

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Example 3: CV Trading Post

The Trading Post on Grand Mesa is a small, family-run store in a remote part of Colorado. The Grand Mesa region contains many good fishing lakes, so the Trading Post sells spinners (a type of fishing lure). The store has a very limited selected of spinners. In fact, the Trading Post has only eight different types of spinners for sale. The prices (in dollars) are

2.10 1.95 2.60 2.00 1.85 2.25 2.15 2.25

Since the Trading Post has only eight different kinds of spinners for sale, we consider the eight data values to be the *population.*

a) Use a calculator to determine the μ and σ.

$\mu = 2.144 \quad \sigma = .217$

b) Compute the CV of prices for the Trading Post.

```
1-Var Stats
x̄=2.14375
Σx=17.15
Σx²=37.1425
Sx=.2321291205
σx=.2171369096
n=8
.217/2.144
          .1012126866
```

```
L1    L2    L3    1
2.6
2
1.85
2.25
2.15
2.25
L1(9)=
```

$CV = \frac{\sigma}{\mu} \cdot 100\% = \frac{.217}{2.144}(100)$ $\boxed{10.121\%}$

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Practice: CV Cabela's

Cabela's in Sidney, Nebraska, is a very large outfitter that carries a broad selection of fishing tackle. It markets its products nationwide through a catalog service. A random sample of 10 spinners from Cabela's extensive spring catalog gave the following prices (in dollars):

1.69 1.49 3.09 1.79 1.39 2.89 1.49 1.39 1.49 1.99

Calculate the mean and standard deviation for this data. Then compute the CV.

```
L1    L2    L3    1
1.39
2.89
1.49
1.39
1.49
1.99
L1(11) =
```

```
1-Var Stats
x̄=1.87
Σx=18.7
Σx²=38.445
Sx=.6214677966
σx=.5895761189
↓n=10
```

$\bar{x} = 1.87$

$S_x = .621$

$CV = \frac{s}{\bar{x}} \cdot 100\% = \frac{.621}{1.87}(100) \approx \boxed{33.209\%}$

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HW: pg. 111: 1, 3, 5, 17

1. Mean

3. Yes, for standard deviation, s, the sum is divided by n - 1, where n is the sample size. For the population standard deviation, sigma, the sum is divided by N, where N is the population size.

5. a) range is 4. b) s ≈ 1.58 c) σ ≈ 1.41

17. a) 7.87 b) used the calculator c) $\bar{x} \approx 1.25$; $s^2 \approx 178$; s ≈ 1.33

d) CV = 107 %. The standard deviation of the time to fairlure is just slightly larger than the average time.

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