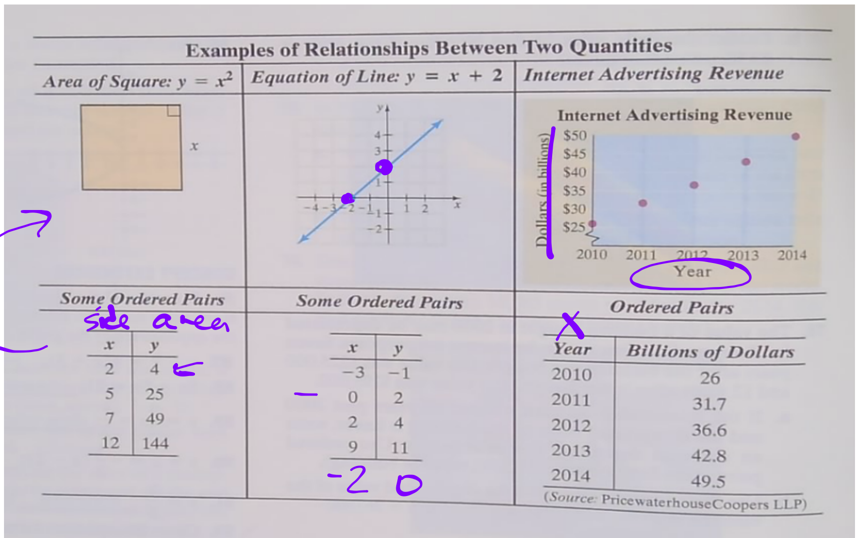
3.6 Functions

OBJECTIVE 1: Identifying Relations, Domains, & Ranges



Examples of Relationships Between Two Quantities

Area of Square: $y = x^2$	*Equation of Line:* $y = x + 2$	*Internet Advertising Revenue*

Some Ordered Pairs

x	y
2	4
5	25
7	49
12	144

Some Ordered Pairs

x	y
−3	−1
0	2
2	4
9	11

Ordered Pairs

Year	*Billions of Dollars*
2010	26
2011	31.7
2012	36.6
2013	42.8
2014	49.5

(*Source:* PricewaterhouseCoopers LLP)

A set of ordered pairs is called a relation. The set of all x-coordinates is called the domain of a relation, and the set of all y-coordinates is called the range of a relation.

Example 1: Find the domain and the range of the relation: {(0, 2), (3, 3), (-1, 0), (3, -2)}. y ↑

D: {-1, 0, 3}

R: {-2, 0, 2, 3}

Practice 1: Find the domain and the range of the relation: {(1, 3), (5, 0), (0, -2), (5, 4)}.

D: {0, 1, 5}

R: {-2, 0, 3, 4}

OBJECTIVE 2: Identifying Functions

SOME relations are also FUNCTIONS.

Function

A function is a set of ordered pairs that assigns to each *x*-value exactly one *y*-value.

Example 2: Determine whether each relation is also a function.

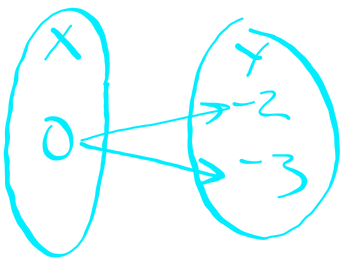
a) {(-1, 1), (2, 3), (7, 3), (8, 6)}

function

b) {(0, -2), (1, 5), (0, 3), (7, 7)}

not a function

(0, -2)
(0, 3)



Practice 2: Determine whether each relation is also a function.

a) {(4, 1), (3, -2), (8, 5), (-5, 3)}

function

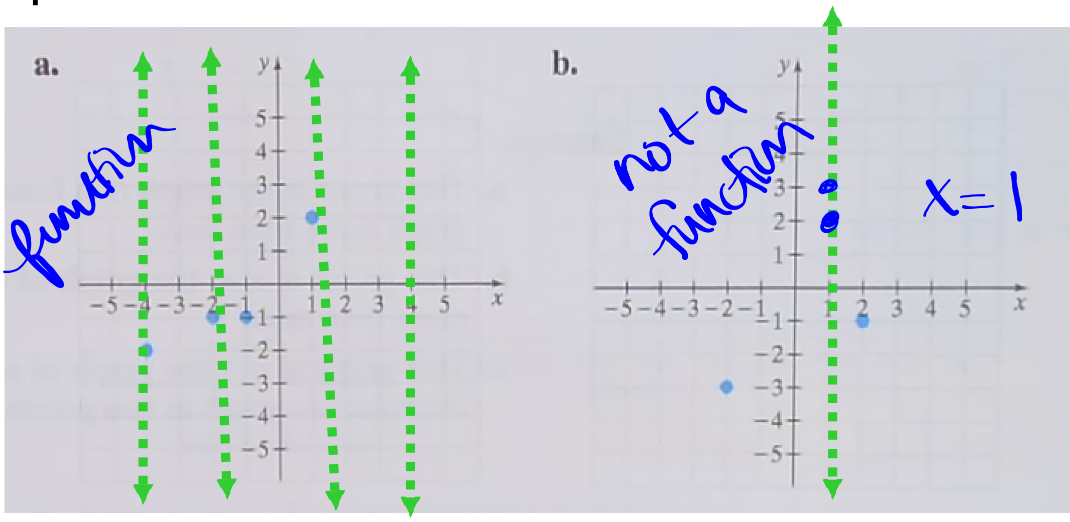
b) {(1, 2), (-4, 3), (0, 8), (1, 4)}

1 → 2, 4 not a function

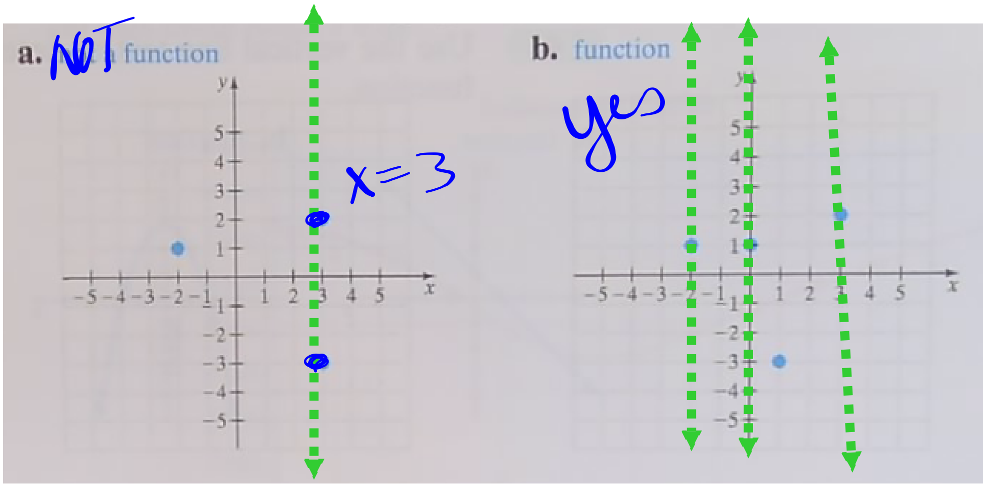
OBJECTIVE 3: Using the Vertical Line Test

When an x-coordinate is paired with more than one y-coordinate, a vertical line can be drawn that will intersect the graph at more than one point. We can use this fact to determine whether a relation is also a function. We call this the vertical line test.

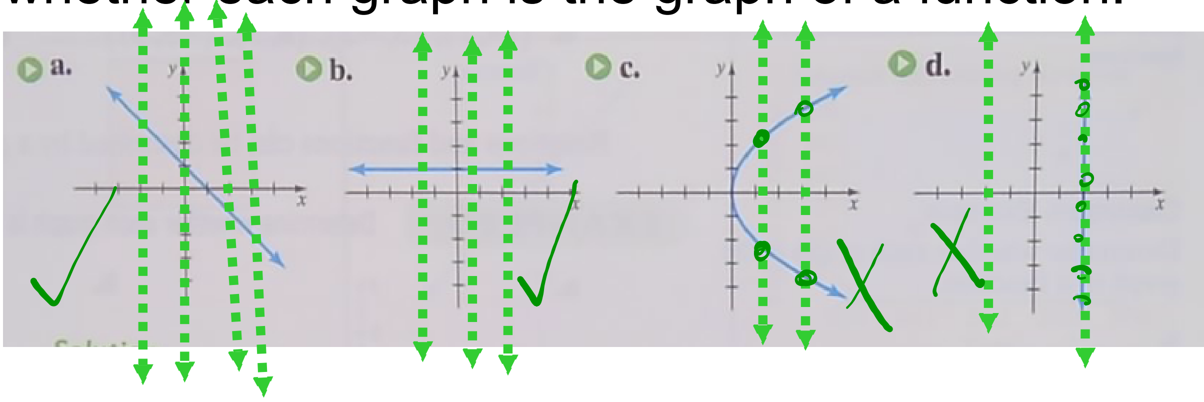
Example 3: Determine whether each graph is the graph of a function.



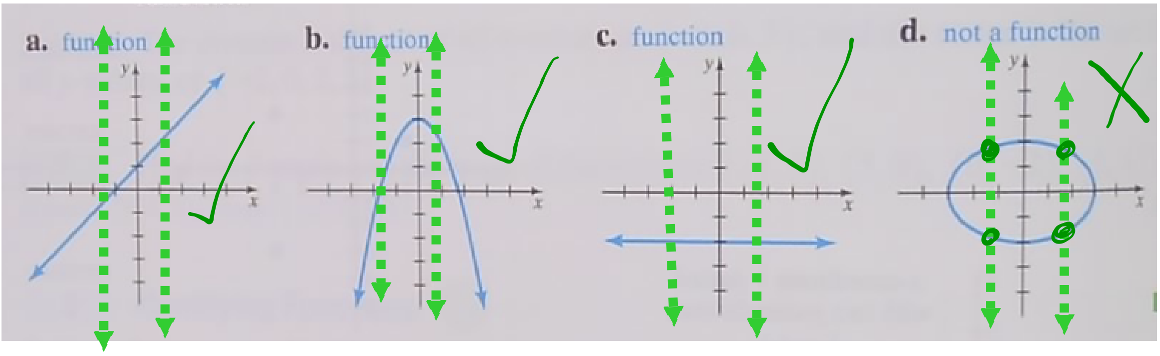
Practice 3: Determine whether each graph is the graph of a function.



Example 4: Use the vertical line test to determine whether each graph is the graph of a function.

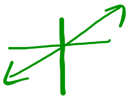


Practice 4: Use the vertical line test to determine whether each graph is the graph of a function.

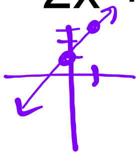


Example 5: Decide whether the equation describes a function.

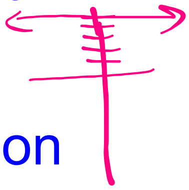
function | not a function

a) $y = x$ 

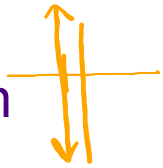
function

b) $y = 2x + 1$ 

function

c) $y = 5$ 

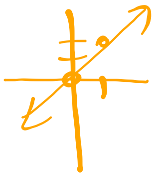
function

d) $x = -1$ 

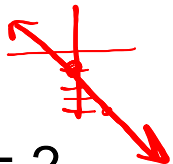
not a function

Practice 5: Decide whether the equation describes a function.

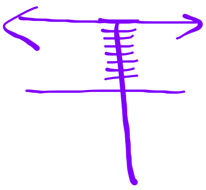
function | not a function

a) $y = 2x$ 

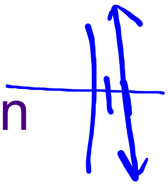
function

b) $y = -3x - 1$ 

function

c) $y = 8$ 

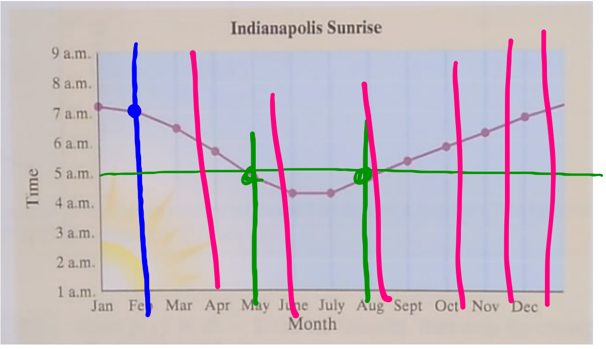
function

d) $x = 2$ 

not a function

Examples of functions can often be found in magazines, newspapers, books, and other printed material in the form of tables or graphs such as the one in the next example.

Example 6: The graph shows the sunrise time for Indianapolis, Indiana, for the year. Use this graph to answer the questions below.



a) Approximate the time of sunrise on Feb 1.

7:01 a.m.

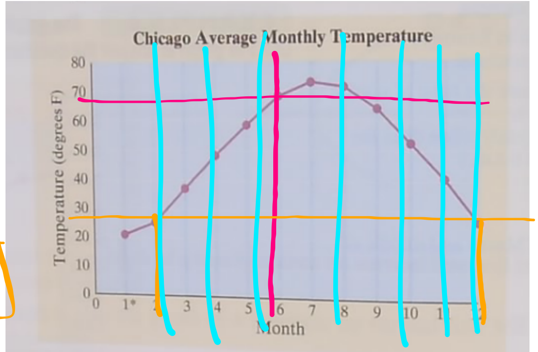
b) Approximate when does the sun rise at 5 a.m.?

May & Aug

c) Is this graph of a function?

Yes

Practice 6: The graph shows the average monthly temperature for Chicago, Illinois, for this year. Use this graph to answer the questions below.



a) Approximate the average monthly temperature for June.

68°F

b) For what month is the average monthly temperature 25°?

2nd & 12th ⇒ Feb & Dec

c) Is this graph of a function?

yes

OBJECTIVE 4: Using Function Notation

The graph of the linear equation y = 2x + 1 passes the *vertical line test*, so we know it is a function. So, y = 2x + 1 gives us a rule for writing ordered pairs where **every x-coordinate is paired with one and only one y-coordinate**.

The variable y is a function of the variable x. **For each value of x, there is only one value of y**. Thus, we say the variable x is the independent variable because any value in the domain can be assigned to x. The variable y is the dependent variable because its value depends on x.

We often use letters such as *f, g,* and *h* to name functions. For example, the symbol *f(x)* means *function of x* and is read "*f of x*." This notation is called function notation.

g(x) h(x)

So... y = 2x + 1 can be written as f(x) = 2x + 1.

f(1) means to replace all x's with 1.

f(x) f(1)
x=1

So... f(1) = 2(1) + 1 simplifies to be 3.

So, f(1) = 3 or (1, 3) as a coordinate.

x y

Example 7:

Given $g(x) = x^2 - 3$, find the following. Then write the corresponding ordered pairs generated.

a) g(2) $x=2$

$g(2) = (2)^2 - 3$
$= 4 - 3$
$g(2) = 1$
$(2, 1)$

b) g(-2) $x=-2$

$g(-2) = (-2)^2 - 3$
$= 4 - 3$
$g(-2) = 1$
$(-2, 1)$

c) g(0) $x=0$

$g(0) = (0)^2 - 3$
$= 0 - 3$
$g(0) = -3$
$(0, -3)$

Practice 7:

Given $h(x) = x^2 + 5$, find the following. Then write the corresponding ordered pairs generated.

a) h(2) $x = 2$

$h(2) = (2)^2 + 5$
$= 4 + 5$
$h(2) = 9$
$(2, 9)$

b) h(-5) $x = -5$

$h(-5) = (-5)^2 + 5$
$= 25 + 5$
$h(-5) = 30$
$(-5, 30)$

c) h(0) $x = 0$

$h(0) = (0)^2 + 5$
$= 0 + 5$
$h(0) = 5$
$(0, 5)$

Example 8: Find the domain of each function.

Hint: Can we divide by 0?

a) $g(x) = \frac{1}{x}$ $\mathbb{R}$ $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

b) f(x) = 2x + 1

$\mathbb{R}$; $(-\infty, \infty)$

Practice 8: Find the domain of each function.

a) h(x) = 6x + 3

$\mathbb{R}$ $(-\infty, \infty)$

b) $f(x) = \frac{1}{x^2}$

$x \neq 0$ $(-\infty, 0) \cup (0, \infty)$

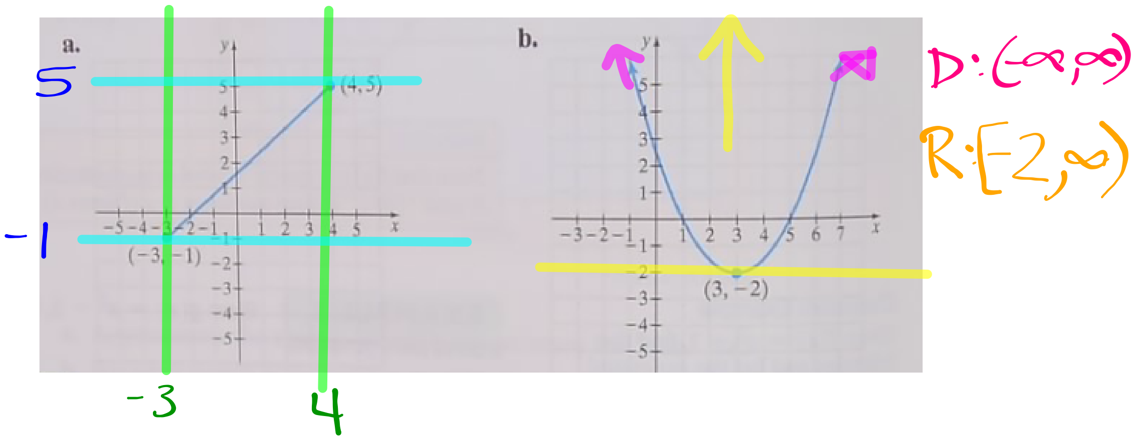
✔ CONCEPT CHECK
Suppose that the value of f is -7 when the function is evaluated at 2. Write this situation in function notation.

$x=2$

$f(2) = -7$

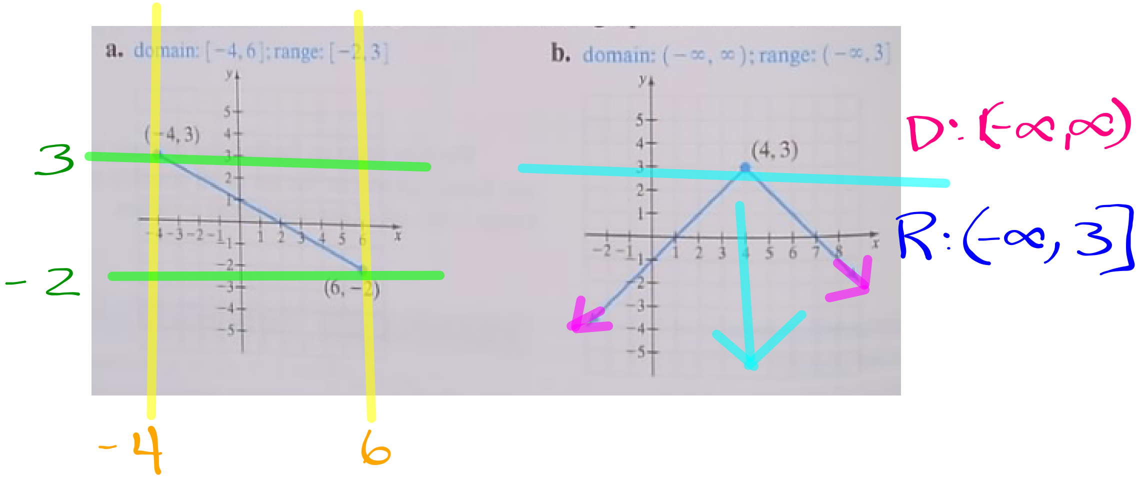
Example 9: Find the domain and the range of each function graphed. Use interval notation.

x → y ↑



D: [−3, 4] R: [−1, 5]

Practice 9: Find the domain and the range of each function graphed. Use interval notation.



D: [−4, 6] R: [−2, 3]

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may not be used.

$x = c$	horizontal	domain	relation
$y = c$	vertical	range	function

1. A set of ordered pairs is called a(n) relation.
2. A set of ordered pairs that assigns to each x-value exactly one y-value is called a(n) function.
3. The set of all y-coordinates of a relation is called the range.
4. The set of all x-coordinates of a relation is called the domain.
5. All linear equations are functions except those whose graphs are vertical lines.
6. All linear equations are functions except those whose equations are of the form $x = c$.

HW: pg. 237

1, 5, 7, 9, 11, 15, 17, 21-37(o), 45, 51 - 81 (o)