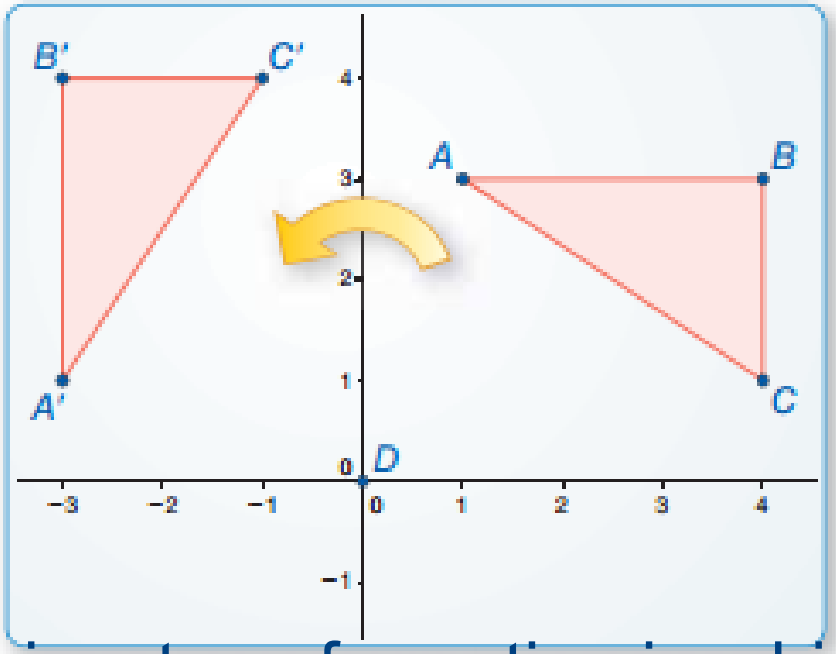
4.3 Rotations

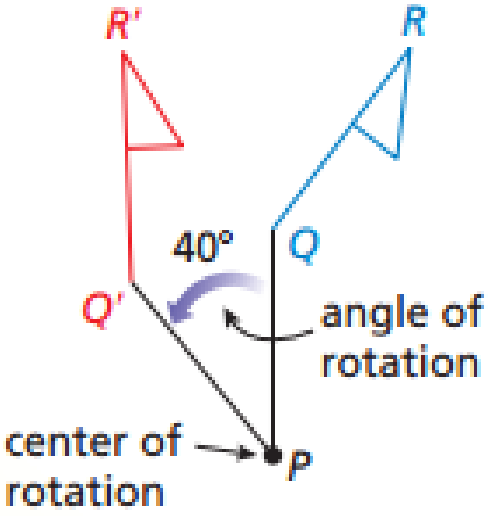


turn

A rotation is a transformation in which a figure is turned about a fixed point called the center of rotation. Rays drawn from the center of rotation to a point and its image form the angle of rotation.

A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q' so that one of the following properties is true.

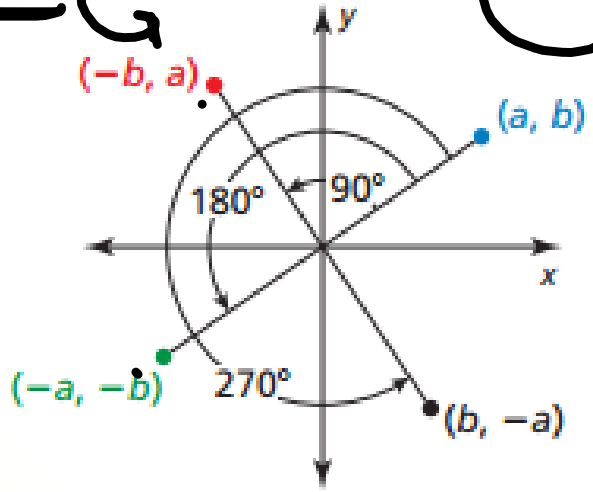
- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then $Q = Q'$.



Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90°, $(x, y) \rightarrow (-y, x)$.
- For a rotation of 180°, $(x, y) \rightarrow (-x, -y)$.
- For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.



Examples: Rotating a Figure in the Coordinate Plane

Graph quadrilateral RSTU with vertices R(3, 1), S(5, 1), T(5, -3), and U(2, -1) and its image after a 270° rotation about the origin.

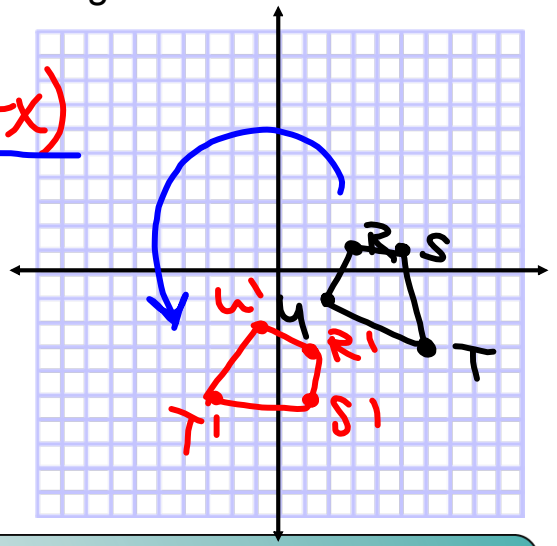
270° $(x, y) \rightarrow (y, -x)$

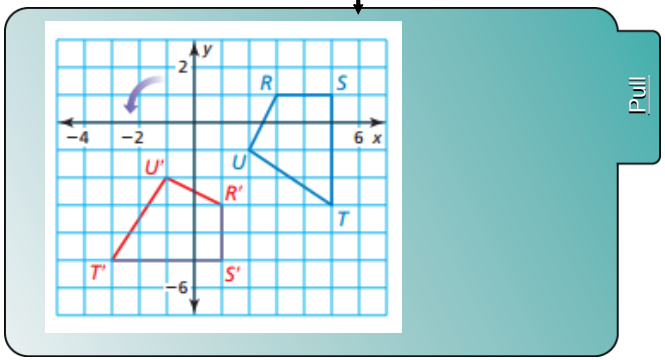
R' $(1, -3)$

S' $(1, -5)$

T' $(-3, -5)$

U' $(-1, -2)$





Examples: Rotating a Figure in the Coordinate Plane

Graph quadrilateral RSTU with vertices R(3, 1), S(5, 1), T(5, -3), and U(2, -1) and its image after a 180° rotation about the origin.

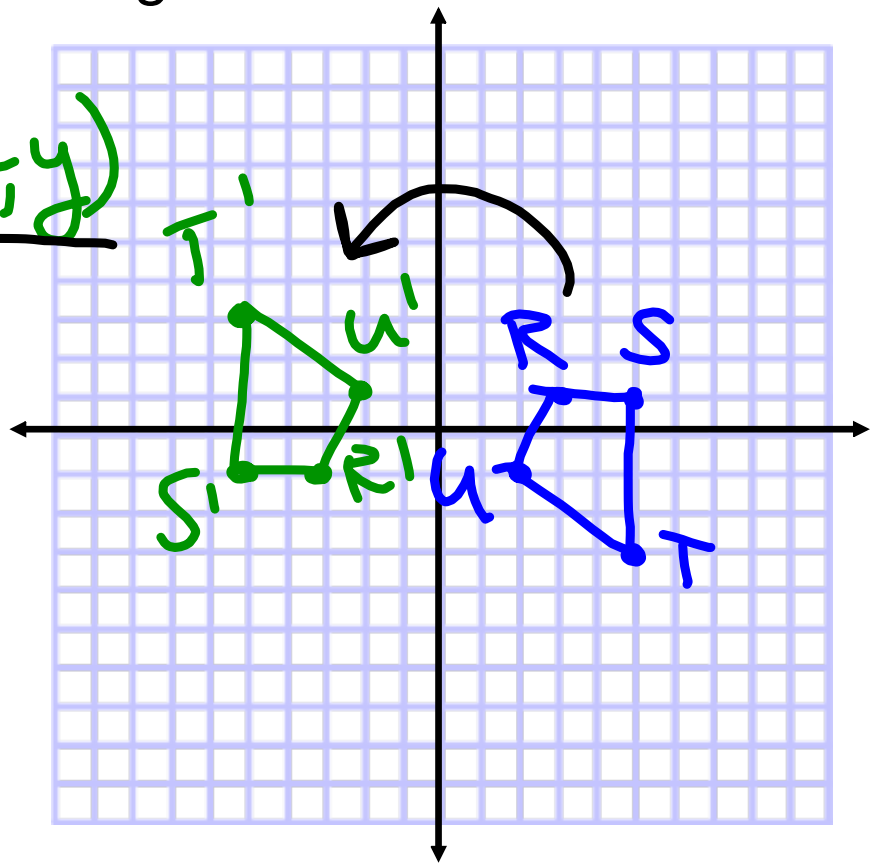
180° $(x, y) \rightarrow (-x, -y)$

R' (-3, -1)

S' (-5, -1)

T' (-5, +3)

U' (-2, 1)



Practice: Rotating a Figure in the Coordinate Plane

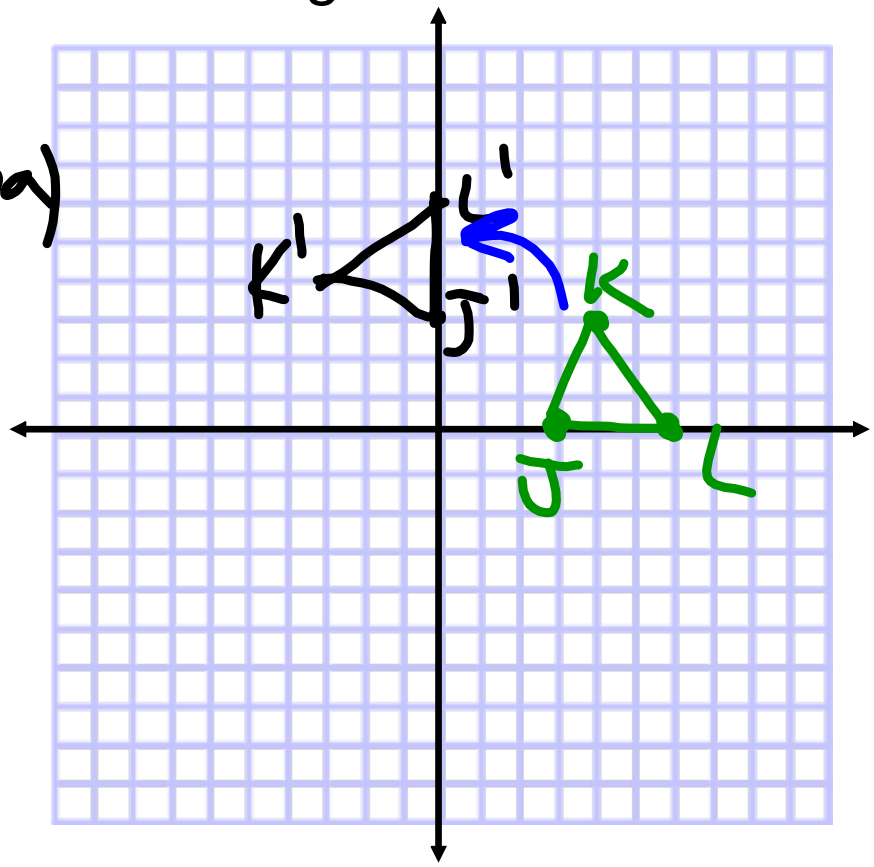
Graph ΔJKL with vertices J(3, 0), K(4, 3), & L(6, 0) and its image after a 90° rotation about the origin.

$90^\circ (a, b) \rightarrow (-b, a)$

$J'(0, 3)$

$K'(-3, 4)$

$L'(0, 6)$



Practice: Rotating a Figure in the Coordinate Plane

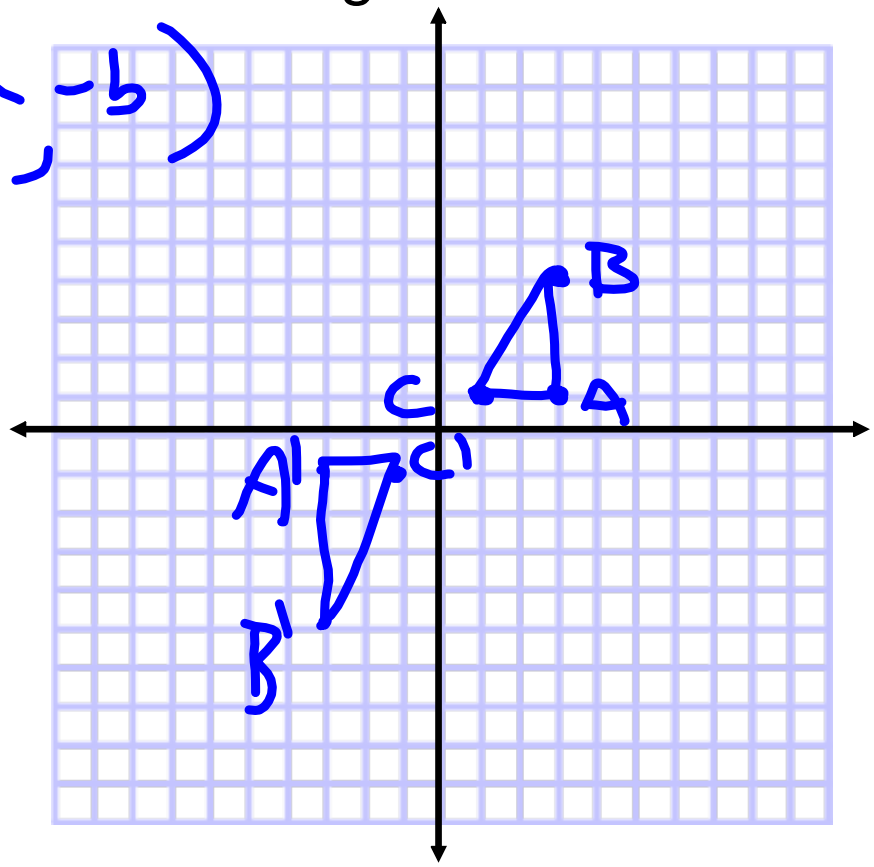
Graph ΔABC with vertices A(3, 1), B(3, 4), & C(1, 1) and its image after a 180° rotation about the origin.

180° $(a, b) \rightarrow (-a, -b)$

$A'(-3, -1)$

$B'(-3, -4)$

$C'(-1, -1)$



Examples: Composition of Transformations

Graph $\overline{RS}$ with endpoints R(1, -3) and S(2, -6) and its image after the composition.

Reflection: over the y- axis

Rotation: 90° about the origin

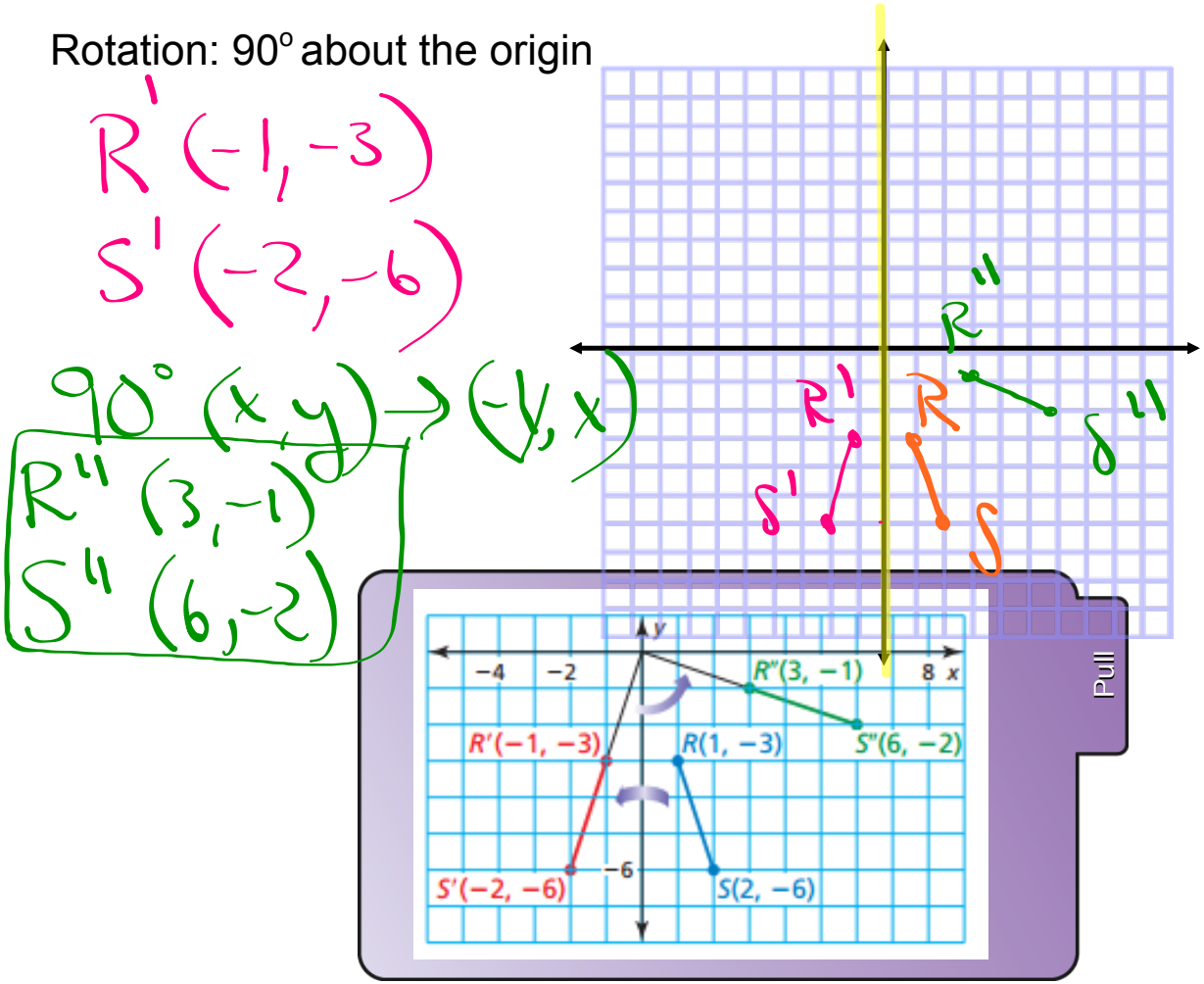
$R'(-1,-3)$

$S'(-2,-6)$

$90°\ (x,y) \rightarrow (-y,x)$

$R''(3,-1)$

$S''(6,-2)$



Practice: Performing a Composition

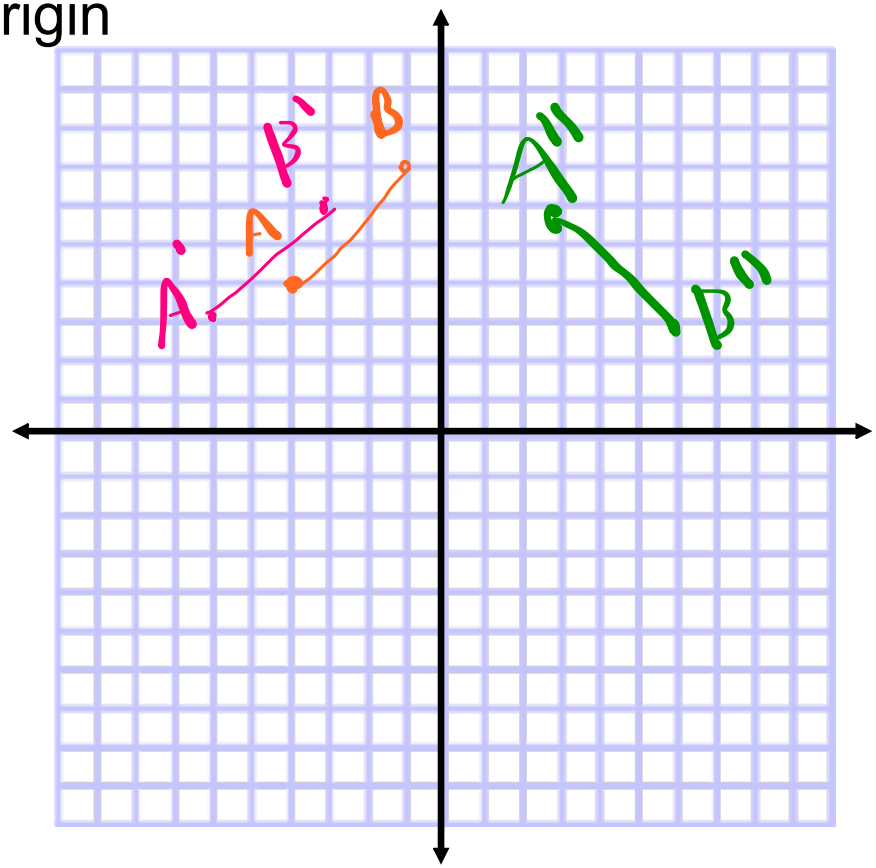
Graph $\overline{AB}$ with endpoints A(-4, 4) & B(-1, 7) & its image after the composition.

Translation: (x, y) ⇒ (x - 2, y - 1)

Rotation: 270° about the origin

A' (-6, 3)
B' (-3, 6)
(x, y) → (y, -x)

A'' (3, 6)
B'' (6, 3)



ACT Practice:

If m = 4, n = -5, and p = 9, what is the value of mp - mn?

A. 16

B. 31

C. 41

D. 56

E. 81

$4(9)-(4)(-5)$

$36-(-20)$

$36+20$

HW: pg. 194: 7, 9, 11, 13, 15, 25, 40 - 41