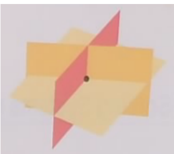
4.4 Solving Systems of Linear Equations in Three Variables

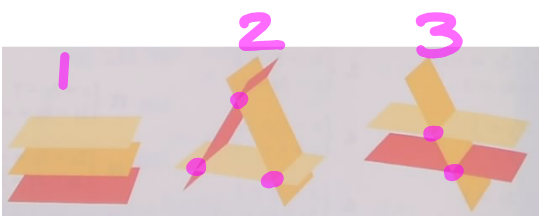
We still use **elimination/additon method**, but now there are three variables like this: $3x - y + z = -15$ so we need three equations to solve. A **linear equation in three variables** and each variable only has a power of 1. A solution of this equation is an **ordered triple (x, y, z) or (a, b, c)** typically written in alphabetical order. Same rules apply for your solution being a true statement in all three equations and for checking your solution in all three equations!

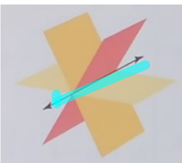
One difference is the visual aspect of the three planes and how they can intersect to give you different types of solutions.



A consistent system that has one ordered triple as its solution.

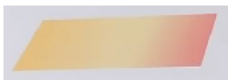
An inconsistent solution to the system shows a few ways it can happen which means there is no solution, not ONE common point of intersection.





A consistent system where all three planes intersect in a single line means there are infinitely many solutions. All points are intersections.

Consistent and dependent because the planes all coincide.



OBJECTIVE 1: Solving a System of Three Linear Equations in Three Variables.

Similar to the systems for the quiz, you need to use elimination to get rid of one variable TWICE. 3 $\frac{1}{2} \times$ (4) $\frac{3}{1} \times$ (5)

Then, you have two variables and two equations and you can use substitution or elimination to find the solutions for one of your variables. $\frac{4}{5}$ $y \Longrightarrow z = \#$

Next, use the same steps to plug back in to determine the second variable. $\frac{4}{5}$ $z \Longrightarrow y = \#\ ;\ z = \#$

Finally, plug all three variables back in to know all three variables to create your ordered triple. $\frac{1}{2}{3}$ y & $z \Longrightarrow x = \#$

(x, y, z)

Example 1: Solve the system.

$$\begin{cases} 3x - y + z = -15 & (1) \\ x + 2y - z = 1 & (2) \\ 2x + 3y - 2z = 0 & (3) \end{cases}$$

(2)

1 $3x - y + \cancel{z} = -15$
2 $x + 2y - \cancel{z} = 1$
(4) $4x + y = -14$

3 $2x + 3y - 2\cancel{z} = 0$
$6x - 2y + 2\cancel{z} = -30$
(5) $8x + y = -30$
(-1)

4 $4x + \cancel{y} = -14$
$-8x - \cancel{y} = 30$
$-4x = 16$
$\frac{-4x}{-4} = \frac{16}{-4}$
$\boxed{x = 4}$

$4(-4) + y = -14$
$-16 + y = -14$
$+16 \qquad +16$
$\boxed{y = 2}$

2) $-4 + 2(2) - z = 1$
$-4 + 4$
$\frac{-z}{-1} = \frac{1}{-1}$
$\boxed{z = -1}$

$$\boxed{(-4, 2, -1)}$$

```
3(-4)-(2)+(-1)
             -15
-4+2(2)-(-1)
               1
2(-4)+3(2)-2(-1)
               0
```

Practice 1: Solve the system.

$$\begin{cases} 3x + 2y - z = 0 & (1) \\ x - y + 5z = 2 & (2) \\ 2x + 3y + 3z = 7 & (3) \end{cases}$$

(2)(3)

1) $3x + \cancel{2y} - z = 0$
2) $2x - \cancel{2y} + 10z = 4$
④ $5x + 9z = 4$

3) $2x + \cancel{3y} + 3z = 7$
2) $3x - \cancel{3y} + 15z = 6$
⑤ $5x + 18z = 13$

(-1)
4) $-\cancel{5x} - 9z = -4$
5) $\cancel{5x} + 18z = 13$
$\cancel{9}z = 9$, $\frac{9}{9}$
$\boxed{z = 1}$

$5x + 9(1) = 4$
$5x + \cancel{9} = 4$
$-9 \quad -9$
$\cancel{5}x = -5$, $\frac{-5}{5}$
$\boxed{x = -1}$

2) $-1 - y + 5(1) = 2$
$-1 - y + 5 = 2$
$\cancel{4} - y = 2$
$-4 \quad -4$
$\cancel{-}y = -2$, $\frac{-2}{-1}$
$\boxed{y = 2}$

$\boxed{(-1, 2, 1)}$

```
-1-2+5(1)
                  2
2(-1)+3(2)+3(1)
                  7
2(-1)+3(2)+3(1)
                  7
```

Example 2: Checking the solution to the system.

$$\begin{cases} x + y - z = -1 \; ✓ \\ -4x - y + 2z = -7 \; ✓ \\ 2x - 2y - 5z = 7 \; ✓ \end{cases}$$

```
3-3-1
                  -1 ✓
-4(3)-(-3)+2(1)
                  -7 ✓
2(3)-2(-3)-5(1)
                  7 ✓
```

(3, -3, 1)

yep this is a solution.

Practice 2: Checking the solution to the system.

$$\begin{cases} x - y + z = -4 \\ 3x + 2y - z = 5 \\ -2x + 3y - z = 15 \end{cases}$$

$-1 - 5 + 2 = -4$

$3(-1)+2(5)-2$

5 ✓

$3(-1)+2(5)-2$

5 ✓

$-2(-1)+3(5)-2$

15 ✓

(-1, 5, 2)

Yes this is a solution.

Worksheet 4.4