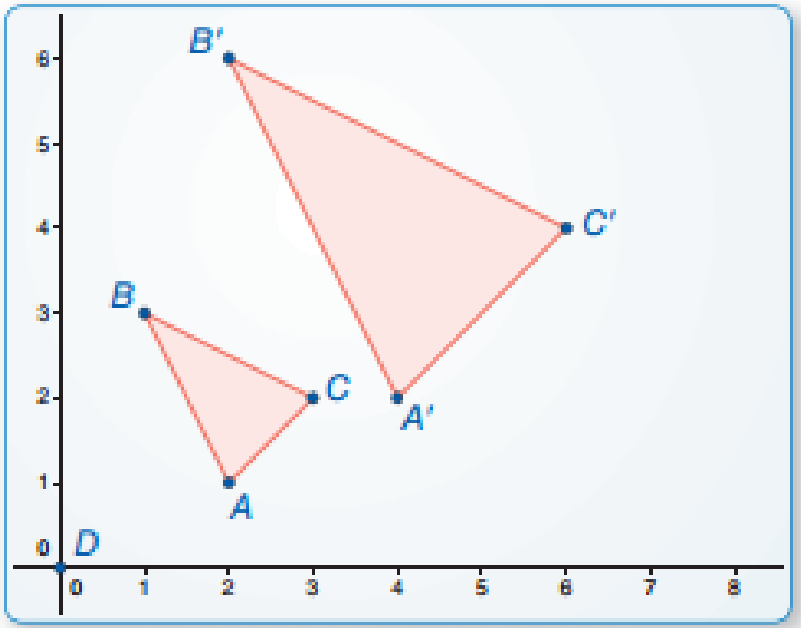
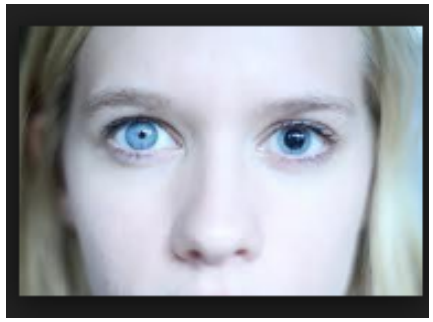
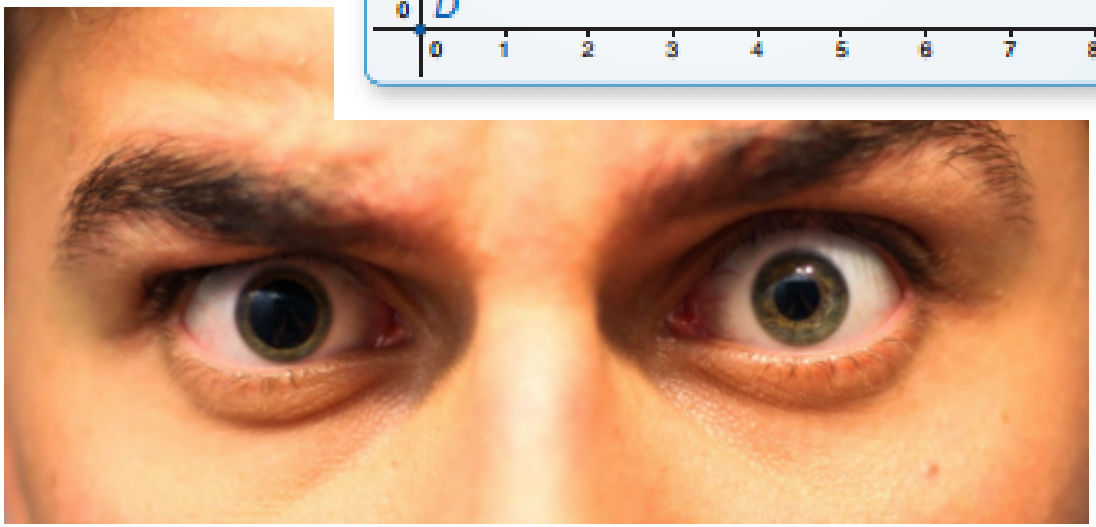
4.5 Dilations





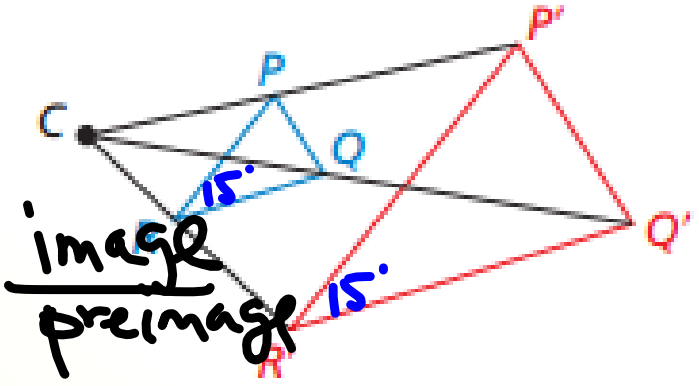


Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.



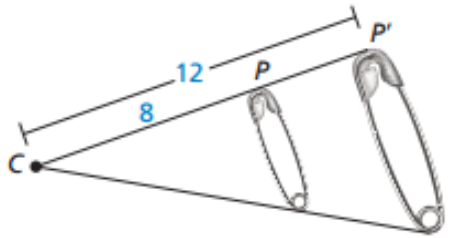
new / old = image / preimage

When the scale factor ($k > 1$) it is an enlargement.
When the scale factor ($0 < k < 1$) it is a reduction.

Example: Identifying Dilations

Find the scale factor (k) of the dilation. Then tell if it is a reduction or an enlargement.

1.

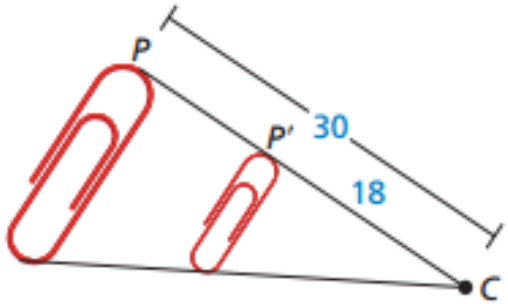


$CP = 8$

$CP' = 12$

$k = \frac{12}{8} = \frac{3}{2}$

2.



$CP = 30$

$CP' = 18$

$k = \frac{18}{30} = \frac{3}{5}$

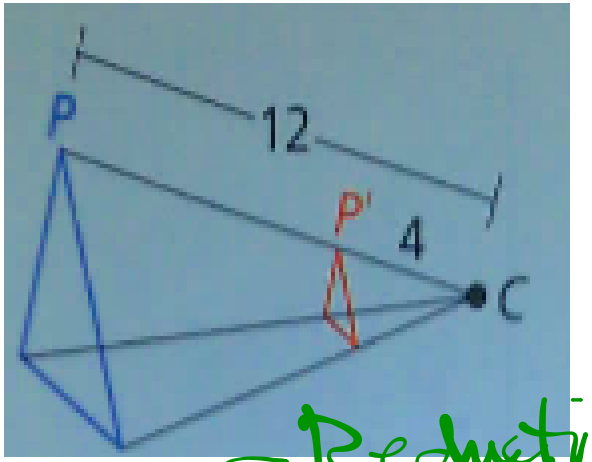
Practice: Identify the Dilation

1. CP' = 3 and CP = 12. Find the scale factor. Then then decide *enlargement* or *reduction.*

$k = \frac{3}{12} = \boxed{\frac{1}{4}}$

2.

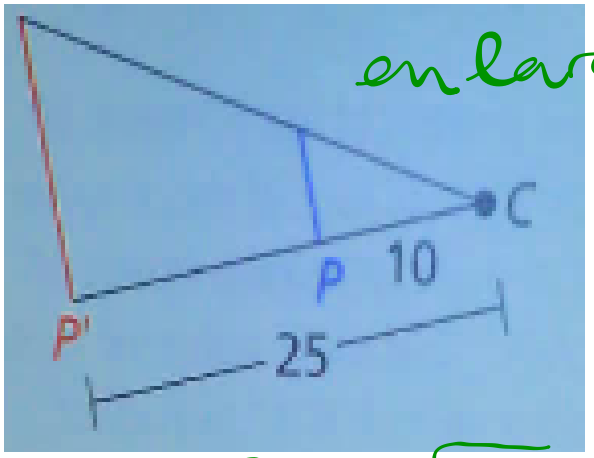
a)



Reduction

$k = \frac{4}{12} = \boxed{\frac{1}{3}}$

b)



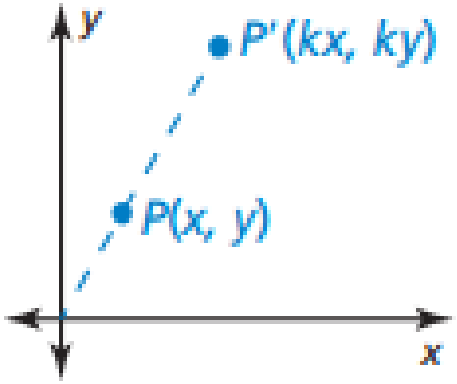
enlargement

$k = \frac{25}{10} = \boxed{\frac{5}{2}}$

Coordinate Rules for Dilations

If $P(x, y)$ is the preimage of a point, then its image P' after a dilation centered at C with scale factor k is shown below.

Center	Image
$(0, 0)$	$P'(kx, ky)$

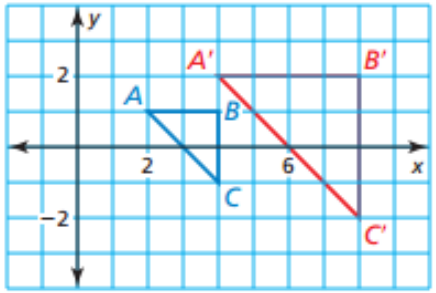


Dilation with center at the origin

$(x, y) \rightarrow (kx, ky)$

Example: Dilating a figure in the coordinate plane

Write the rule for the dilation used.



enlargement

$P'(2x, 2y)$

$A(2,1)$

$A'(4,2)$

$k = 2$

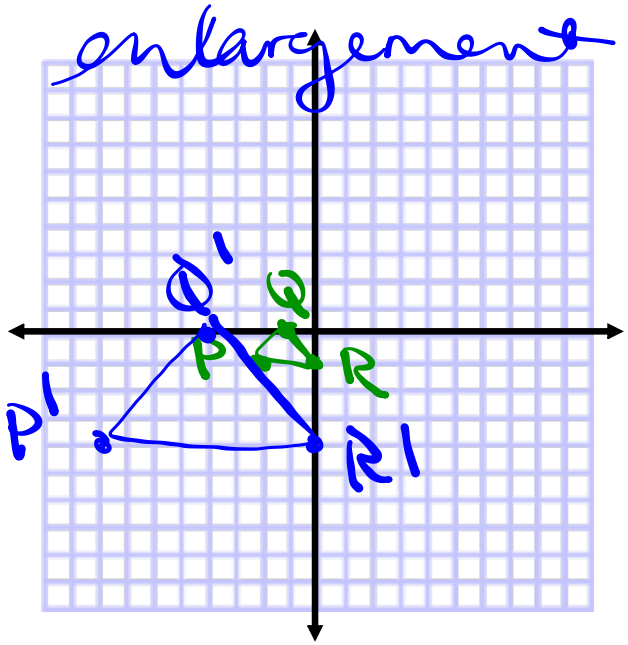
$(x,y) \rightarrow (2x, 2y)$

Practice: Dilating a figure in the coordinate plane

Graph the image and write the dilation rule given k. Write the final image coordinates.

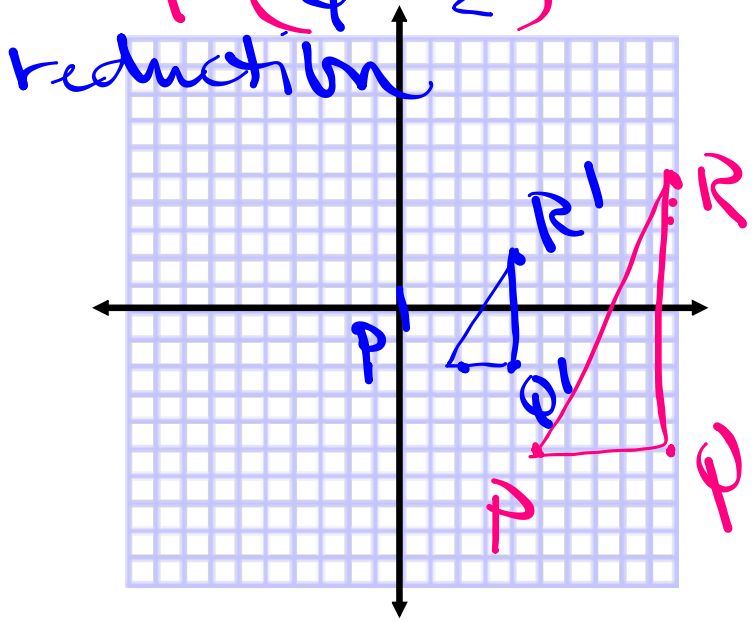
a) P(-2, -1), Q(-1, 0), R(0, -1);
k = 4 $(x,y) \rightarrow (4x, 4y)$

$P'(-8, -4)$ $Q'(-4, 0)$ $R'(0, -4)$



b) P(5, -5), Q(10, -5), R(10, 5);
k =0.4 $(x,y) \rightarrow (.4x, .4y)$

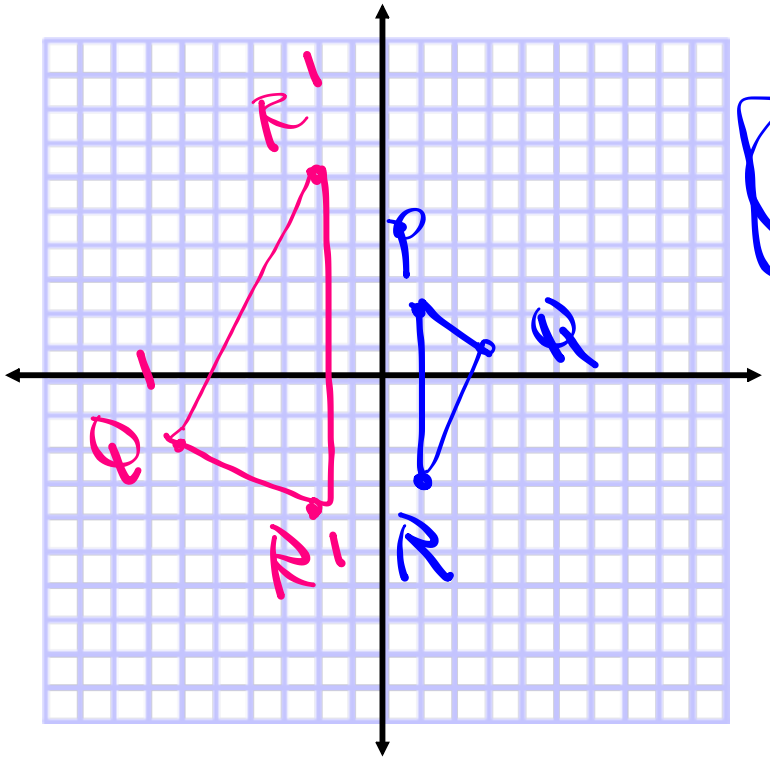
$P'(2, -2)$ $Q'(4, -2)$ $R'(4, 2)$



If your scale factor (k) is negative it is doing two transformations at once. It is rotating 180 degrees and dilating your preimage.

EXAMPLE: Using a Negative Scale Factor

Graph ΔPQR with vertices P(1, 2), Q(3, 1), & R(1, -3) and its image after a dilation with a scale factor of -2.



$k = -2$

$(x, y) \rightarrow (-2x, -2y)$

$P'(-2, -4)$

$Q'(-6, -2)$

$R'(-2, 6)$

Practice: Using a Negative Scale Factor

Graph ΔFGH with vertices F(3, 6), G(3, -3), & H(6, 6) and its image after a dilation with a scale factor of $-\frac{1}{3}$.

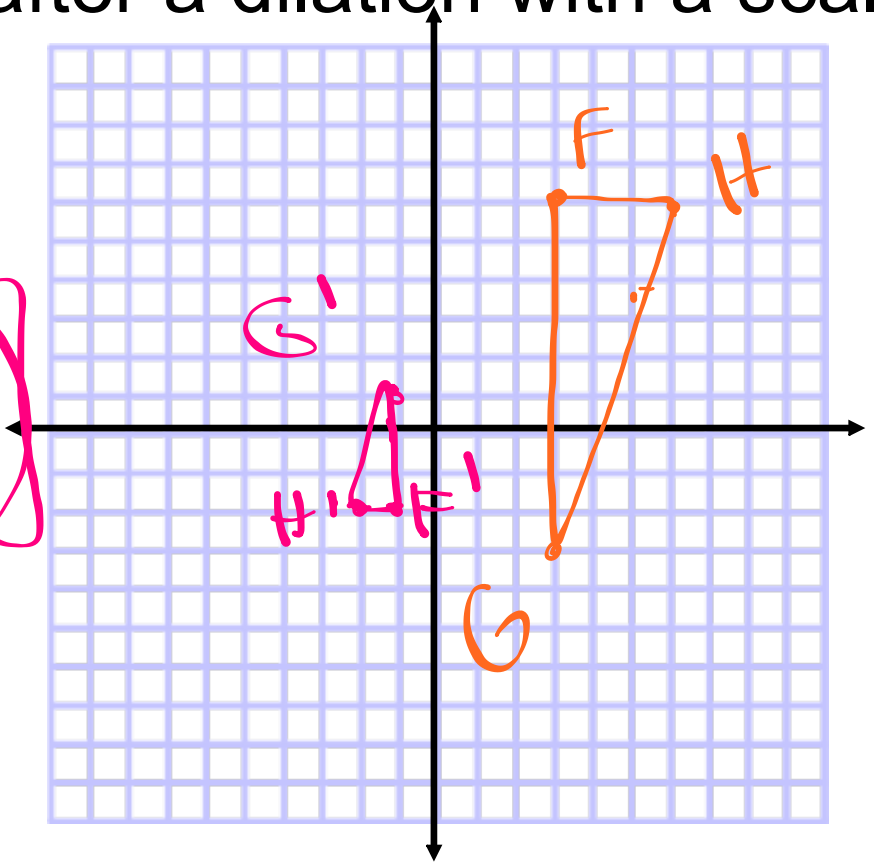
$k = -\frac{1}{3}$

$(x, y) \rightarrow \left(-\frac{1}{3}x, -\frac{1}{3}y\right)$

F' (-1, -2)

G' (-1, 1)

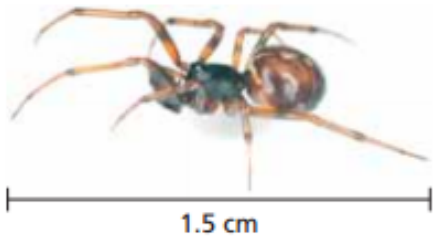
H' (-2, -2)



Example: Solving Real-Life Problems

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?

$k = \frac{new}{old} = \frac{1.1}{4} = .275$

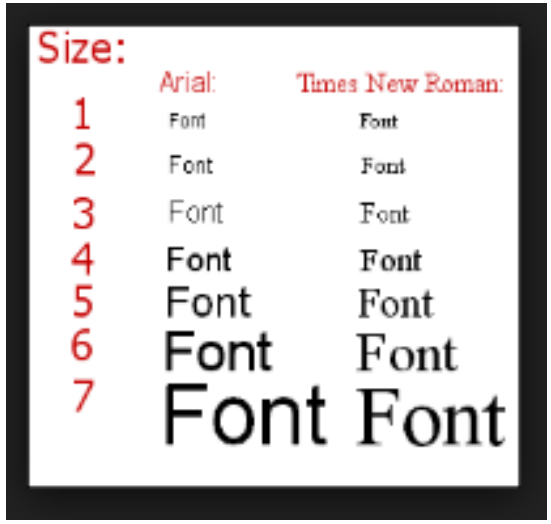


You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.

$k = 6 \quad P = 1.5 \quad P' = 9\text{ cm} \quad \frac{6}{1} = \frac{x}{1.5}$

Practice: Solving Real Life Problems

You are using word processing software to type the outline school newsletter. You change the size of the text in one headline from 0.5 inch tall to 1.25 inches tall. What is the scale factor of this dilation?



$k = \frac{\text{new}}{\text{old}}$

$k = \frac{1.25}{.5} = 2.5$

ACT Practice:

Rectangle ABCD has vertices in the standard (x, y) coordinate plane at A(-4, -2), B(-4, 3), C(2, 3), and D(2, -2). A translation of rectangle ABCD is a second rectangle, A'B'C'D', with vertices A'(4, -12), B'(x , y), C'(10, -7), and D'(10, -12). What are the coordinates of B'?

~~A. (3, -6)~~

B. (4, 3)

C. (4, -7)

D. (4, -13)

~~E. (6, -5)~~

$A \to A' \ (x+8, y-10)$

$B \to B' \ (-4+8, 3-10)$

$C \to C' \ (x+8, y-10)$

$D \to D' \ (x+8, y-10)$

HW: pg. 212: 5, 15, 21, 23, 25, 27, 30, 31, 52 - 57