

Solving a Polynomial Equation by FACTORING

TASK 1:

$$2x^3 - 12x^2 + 18x = 0$$

$$2x(x^2 - 6x + 9) = 0$$

$$2x(x-3)^2 = 0$$

$$2x = 0 \quad (x-3)^2 = 0$$

$$x = 0 \quad x = 3$$

$$x = 0, 3$$

↳ w/ a multiplicity of 2

$$0 = -2x^4 + 16x^2 - 32$$

$$0 = x^4 - 8x^2 + 16$$

$$0 = (x^2 - 4)^2$$

$$0 = x^2 - 4$$

$$4 = x^2$$

$$\pm 2 = x$$

↳ w/ a multiplicity of 2

Rational Root Theorem

TASK 2:

$$f(x) = x^3 - 8x^2 + 11x + 20$$

$$p = 20 \quad q = 1$$

$$\begin{array}{r|l} 1 & 2 & 4 & 5 & 10 & 20 \\ \hline & & & & & 1 \end{array}$$

$$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$$

 $\frac{p}{q} =$ factors of constant term
factors of leading coefficient term

$$p(x) = 3x^3 + x^2 - 14x - 24$$

$$p = -24 \quad q = 3$$

$$\begin{array}{r|l} 1 & 2 & 3 & 4 & 6 & 8 & 12 & 24 \\ \hline & & & & & & & 1 & 3 \end{array}$$

$$\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm 4, \pm \frac{4}{3}, \pm 6, \pm 8, \pm \frac{8}{3}, \pm 12, \pm 24$$

Solving Using the Rational Root Theorem:

STEPS:

- 1) List possible roots ($\frac{p}{q}$)
- 2) Graph the equation on the calculator
- 3) Test possible roots by long/synthetic division
- 4) Factor after finding roots
 - 2 terms: special formulas or GCF
 - 3 terms: BIG X
 - 4 terms: Factor by Grouping
- 5) List the final real solutions as $x = \dots$

TASK 3:

$$x^2 + 12x + 20 = 0$$

$$1) \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$$

$$2) \begin{array}{r} 2 \overline{) 1 \ 12 \ 20} \\ \underline{+ \ 2 \ 24} \\ 1 \ 14 \ 48 \end{array}$$

$$3) x + 10 = 0$$

$$x = -10$$

$$4) x = -2, -10$$

TASK 4: Write the simplest polynomial with roots...

$$-1, \frac{2}{3}, \text{ and } +\sqrt{5}$$

$$x = -1 \quad x = \frac{2}{3} \quad x = \pm\sqrt{5}$$

$$(x+1)(x-\frac{2}{3})(x+\sqrt{5})(x-\sqrt{5})$$

$$(3x^2 + x - 2)(x^2 - 5)$$

$$y = 3x^4 + x^3 - 17x^2 - 5x + 10$$

Still need help with:

The Irrational Conjugates Theorem

If $a + \sqrt{b}$ is a root then automatically $a - \sqrt{b}$ is guaranteed to be another root.

$$x^3 + x^2 - 14x - 24 = 0$$

$$1) \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$

$$2) \begin{array}{r} -3 \overline{) 1 \ 1 \ -14 \ -24} \\ \underline{+ \ 3 \ 6 \ 24} \\ 4 \ 1 \ -2 \ -8 \ 8 \end{array}$$

$$3) x + 2 = 0$$

$$x = -2$$

$$4) x = -3, 4, -2$$

$$x = 3 \text{ and } -\sqrt{3}$$

$$(x-3)(x+\sqrt{3})(x-\sqrt{3})$$

$$(x-3)(x^2-3)$$

$$y = x^3 - 3x^2 - 3x + 9$$