

4.5 Solving Systems of Linear Equations Word Problems

1. Nickels and Dimes: A woman has \$13.00 in nickels and dimes in a coin bank. She finds the number of dimes is 2 less than 5 times the number of nickels. Write and solve a system of equations to determine how many of each kind of coin she has. substitution

Variables:  $x$ : # of nickels  $y$ : # of dimes  $y = 5(24) - 2$   
 $= 120 - 2$   
 $y = 118$

System:  $0.05x + 0.1y = 13$   
 $y = 5x - 2$   
 $0.05x + 0.1(5x - 2) = 13$   
 $0.05x + 0.5x - 0.2 = 13$

The woman has 24 nickels and 118 dimes.

Solution as a complete sentence:  $0.55x - 0.2 = 13$   $x = 24$   
 $0.55x = 13.2$

2. Tea and Coffee: Two pounds of tea and three pounds of coffee cost \$19.00. Three pounds of tea and four pounds of coffee cost \$26.50. Write and solve a system of equations to determine the cost of one pound of tea and one pound of coffee.

Variables:  $x$ : lbs of coffee  $y$ : lbs of tea

System:  $2y + 3x = 19.00$   
 $3y + 4x = 26.50$

3.5 lbs of tea  
 $\frac{1}{2}$  4 lbs of coffee

$6y + 9x = 57$   
 $7y - 8x = 53$   
 $x = 4$   
 $2(y) + 3(4) = 19$   
 $2y + 12 = 19$   
 $2y = 7$   
 $y = 3.5$

Solution as a complete sentence:

3. Thinking of a Number: A father tells his son, "I'm thinking of two numbers." He provided the following clues:

- Twice the first of two numbers is 9 less than the second number.
- Thirteen times the sum of the two numbers is 3 less than the second number.

Find both numbers to know which numbers the father was thinking of.

Variables:  $x$  = 1st #  $y$  = 2nd #

System:  $2x = y - 9$   
 $13(x + y) = y - 3$   
 $13x + 13y = y - 3$

$13x + 12(2x + 9) = -3$   
 $13x + 24x + 108 = -3$   
 $37x + 108 = -3$   
 $37x = -111$   
 $x = -3$

①  $13x + 12y = -3$     ②  $2x + 9 = y$

Solution as a complete sentence:

The father's two numbers are 3 and -3.

$2(-3) = y - 9$   
 $-6 = y - 9$   
 $3 = y$



4. Connie and Walter: Connie and Walter had lunch together at the same stand. Connie paid \$14.40 for her lunch of 4 hamburgers and 3 cokes. Walter paid \$10.50 for 3 hamburgers and 2 cokes. What is the price of one hamburger and one coke?

$x = \text{price of a hamburger}$        $y = \text{price of a coke}$

Connie:  $4x + 3y = 14.40$  (2)  
 Walter:  $3x + 2y = 10.50$  (3)

$4(2.70) + 3y = 14.40$   
 $10.8 + 3y = 14.40$   
 $3y = 3.60$

$y = 1.2$

$-8x - 6y = -28.80$   
 $9x + 6y = 31.50$

$x = 2.70$

A hamburger costs \$2.70 each while a coke is only \$1.20.

5. Towing Company: Auto Shop Towing charges \$0.50 per mile and \$15 to pick you up. Benny's Wrecker Service charges \$0.75 a mile and \$10 to pick you up. Determine when the Auto Shop Towing would cost the same as Benny's Wrecker Service.

$x = \# \text{ of miles}$   
 $y = \text{total cost}$

$0.5x + 15 = 0.75x + 10$   
 $15 = 0.25x + 10$   
 $5 = 0.25x$

$20 = x$

AST:  $y = 0.50x + 15$   
 BWS:  $y = 0.75x + 10$

$y = 0.5(20) + 15$   
 $= 10 + 15$   
 $y = 25$

If you go 20 miles, both companies charge \$25.

6. Planting Trees: Trees in urban areas help keep air fresh by absorbing carbon dioxide. A city has a total of \$2100 to spend on planting spruce and maple trees. The total land available for planting is 45,000 square feet. Spruce trees cost \$30 to plant and require 600 square feet of space. Maple trees cost \$40 to plant and require 900 square feet of space. Spruce trees absorb 650 pounds per year of carbon dioxide. Maple trees absorb 300 pounds per year of carbon dioxide. How many of each tree should the city plant to maximize carbon dioxide absorption?

$x = \# \text{ of spruce trees}$   
 $y = \# \text{ of maple trees}$

$\div 10 \left\{ \begin{array}{l} 30x + 40y = 2100 \\ 600x + 900y = 45,000 \end{array} \right.$

$30x + 40(30) = 2100$   
 $30x + 1200 = 2100$   
 $-1200 \quad -1200$

$30x = 30$   
 $x = 1$

$-2(3x + 4y = 210)$   
 $6x + 9y = 450$   
 $-6x - 8y = -420$

$y = 30$

The city should plant 1 spruce tree & 30 maple trees.