Lesson Title **4.6 The Fundamental Theorem of Algebra Notes** ALG 2 Date ____________

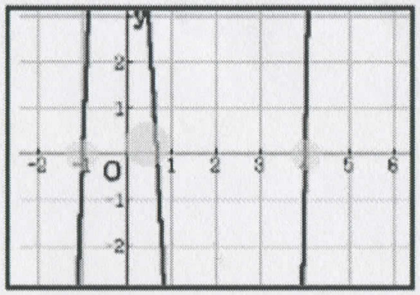
Graph the polynomial below. Notice that the degree of the function is the same as the number of zeros. This is true for all polynomial functions.

Degree = 3

$3x^3 - 11x^2 - 6x + 8$

Number of zeros = 3

$-1, 4, \frac{2}{3}$



However, all of the zeros are not necessarily real zeros.

The degree of a function is the number of times the graph crosses the x-axis.

Find the Zeros of a Polynomial Function:

STEPS:

1) Rational Root Theorem: $\left(\frac{p}{q}\right)$
2) Graph the equation on the calculator
3) Test possible roots by long/synthetic division
4) Factor after finding roots
   - 2 terms: special formulas or GCF
   - 3 terms: BIG X
   - 4 terms: Factor by Grouping
5) List the final real solutions as x = …

The Complex Conjugates Theorem

If you have a + bi then automatically a – bi is also a zero of the same function. Imaginary numbers or complex numbers come in pairs with the ±.

TASK 1: How many solutions does each equation/function have?

$x^3 + 3x^2 + 16x + 48 = 0$ 3

$f(x) = x^4 + 6x^3 + 12x^2 - + 8x$ 4

TASK 2:

a) $f(x) = x^5 + x^3 - 2x^2 - 12x - 8$

1) $\frac{p}{q} = \frac{-8}{1}$: ± 1 ± 2 ± 4 ± 8

2)
2	1	0	1	-2	-12	-8
+	↓	2	4	10	16	8
-1	1	2	5	8	4	$\boxed{0}$ ✓
+	↓	-1	-1	-4	-4	
-1	1	1	4	4	$\boxed{0}$ ✓	
+	↓	-1	0	-4		
	1	0	4	$\boxed{0}$ ✓		

3) $x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4} \Rightarrow x = \pm 2i$

4) $x = 2, -1, \pm 2i$

b) $f(x) = x^5 + 3x^4 + 9x^3 + 23x^2 - 36$

1) $\frac{p}{q} = \frac{-36}{1}$: ± 1 ± 2 ± 3 ± 4 ± 6 ± 9 ± 12 ± 18 ± 36

2)
1	1	3	9	23	0	-36
+	↓	1	4	13	36	36
-2	1	4	13	36	36	$\boxed{0}$ ✓
+	↓	-2	-4	-18	-36	
-2	1	2	9	18	$\boxed{0}$ ✓	
+	↓	-2	0	-18		
	1	0	9	$\boxed{0}$ ✓		

3) $x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

4) $x = 1, -2, \pm 3i$

TASK 3: Write the simplest polynomial function with the zeros of …

5 and $1 + i$

$x = 5$ $x = 1 + i$ $x = 1 - i$

$(x-5)(x-1-i)(x-1+i)$

	x	-1	i
x	x^2	$-x$	ix
-1	$-x$	$+1$	$-i$
$-i$	$-ix$	$+i$	$-i^2$

-3 and $1 + i\sqrt{5}$

$x = -3$ $x = -1 + i\sqrt{5}$ $x = -1 - i\sqrt{5}$

$(x+3)(x+1-i\sqrt{5})(x+1+i\sqrt{5})$

	x	-1	$i\sqrt{5}$
x	x^2	$-x$	$ix\sqrt{5}$
-1	$-x$	$+1$	$-i\sqrt{5}$
$-i\sqrt{5}$	$-i\sqrt{5}x$	$i\sqrt{5}$	$-i^2 25$

$(x+3)(x^2-2x+6)$

$x^3 - 2x^2 + 6x$

$3x^2 - 6x + 18$

$\boxed{g(x) = x^3 + x^2 + 18}$

Still need help with:

$(x-5)(x^2-2x+2)$

$x^3 - 2x^2 + 2x$

$-5x^2 + 10x - 10$

$\boxed{f(x) = x^3 - 7x^2 + 12x - 10}$