Lesson Title 4.6 The Fundamental Theorem of Algebra DAY TWO Notes **ALG 2 Date** ____________

Descartes' Rule of Signs

Before calculators, this process helped people reduce the guessing and checking possibilities. You count the number of sign changes and create a table in the process. This is done twice, first with f(x) and then with f(- x).

Let P(x) be a polynomial with real coefficients.

- The number of positive zeros of P is either equal to the number of sign changes for P(x) or less by an even number.
- The number of negative real zeros of P is either equal to the number of variations in sign changes of P(- x) or less by an even number.

TASK 1: Create a chart of possibilities.

$f(x) = x^3 + 4x^2 - 25x - 28$

f(x): + + − − 1

f(- x): − + + − 2

3 =

+	-	i
1	2	0
1	0	2

$g(x) = 2x^4 + x^3 + 7x^2 + 4x - 4$

g(x): + + + + − 1

g(- x): + − + − − 3

+	-	i
1	3	0
1	1	2

= 4

$f(x) = x^3 + 4x^2 - 25x - 28$

1) $\frac{p}{q}$: $\frac{-28}{1}$: ±1 ±2 ±4 ±7 ±14 ±28

2) f(x) = + + − − 1

f(-x) = − + + − 2

+	-	i
1	2	0
1	0	2

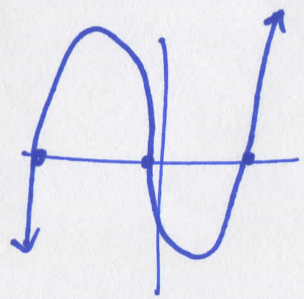
= 3

3)
$$\begin{array}{r|rrrr} 4 & 1 & 4 & -25 & -28 \\ + & \downarrow & 4 & 32 & 28 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

4) $x^2 + 8x + 7 = 0$

$(x+7)(x+1) = 0$

5) $x = -7, -1, 4$



FINDING ZEROS AND FACTORS OF A POLYNOMIAL

- List all possible rational zeros using the Rational Zero Theorem.
- Apply Descartes' Rule of Signs to determine the number of possible positive and negative zeros.
- Check the candidates for possible rational zeros, substituting the values from the smallest in magnitude to the largest.
- When a zero is found, factor the polynomial and repeat the process on the quotient. There is no need to check possible zeros of the quotient that have already been eliminated from the list of zeros at the previous stage. But check again any zeros that have been found at the previous stage since they may have a multiplicity greater than 1.
- If the polynomial has been factored to linear terms and quadratic terms, factor the quadratics, using the quadratic formula if necessary.

FINDING ZEROS

There are several methods you can use to find the zeros of a polynomial function.

- Graphing
- Synthetic Substitution
- Substituting values and using a calculator

$f(x) = 6x^3 + 11x^2 - 3x - 2$

1) $\frac{p}{q} = \frac{-2}{6} : \frac{\pm 1 \quad \pm 2}{\pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 6} = \boxed{\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 2, \pm\frac{2}{3}}$

2) $f(x) = + \; + \; - \; -$ 1
$f(-x) = - \; + \; + \; -$ 2

+	−	i
1	2	3
1	0	2

= 3

3) $-2 \,|\; 6 \quad 11 \quad -3 \quad -2$
$\quad \downarrow \; -12 \quad 2 \quad 2$
$\quad 6 \quad -1 \quad -1 \quad \boxed{0}$ ✓

-6, 2, -3, -1

4) $(6x^2 + 2x)(-3x - 1) = 0$
$2x(3x+1) - 1(3x+1) = 0$
$(2x-1)(3x+1) = 0$

5) $x = -2, \frac{1}{2}, -\frac{1}{3}$

$f(x) = 2x^4 + x^3 + 7x^2 + 4x - 4$

1) $\frac{p}{q} = \frac{-4}{2} : \frac{\pm 1 \; \pm 2 \; \pm 4}{\pm 1 \quad \pm 2} = \boxed{\pm 1, \pm\frac{1}{2}, \pm 2, \pm 4}$

2) $f(x) = + \; + \; + \; + \; -$ 1
$f(-x) = + \; - \; + \; - \; -$ 3

+	−	i
1	3	0
1	1	2

= 4

3) $-1 \,|\; 2 \quad 1 \quad 7 \quad 4 \quad -4$
$\quad \downarrow \; -2 \quad 1 \quad -8 \quad 4$
$\quad 2 \quad -1 \quad 8 \quad -4 \quad \boxed{0}$ ✓

4) $(2x^3 - x^2) + (8x - 4) = 0$
$x^2(2x-1) + 4(2x-1) = 0$
$(x^2+4)(2x-1) = 0$

5) $x = \frac{1}{2}, \pm 2i, -1$

Still need help with: