Lesson Title <u>5.2 Properties of Rational Exponents & Radicals Notes</u> ALG 2 Date ____________________

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

<u>TASK 2:</u> Use properties of exponents to simplify expressions.

a) $2^{\frac{3}{4}} \cdot 2^{\frac{1}{2}}$

$= 2^{\frac{3}{4}+\frac{1}{2}} = 2^{\frac{3}{4}+\frac{2}{4}}$

$\boxed{2^{\frac{5}{4}}}$

b) $\frac{3^{\frac{4}{4}}}{3^{\frac{1}{4}}}$

$= 3^{\frac{4}{4}-\frac{1}{4}} = \boxed{3^{\frac{3}{4}}}$

c) $\left(\frac{20^{\frac{1}{2}}}{5^{\frac{1}{2}}}\right)^3 = \left(\frac{\sqrt{20}}{\sqrt{5}}\right)^3$

$= \left(\sqrt{\frac{20}{5}}\right)^3 = (\sqrt{4})^3$

$= (2)^3 = \boxed{8}$

d) $\left(5^{\frac{1}{3}} \cdot 7^{\frac{1}{4}}\right)^3 = 5^{\frac{3}{3}} \cdot 7^{\frac{3}{4}}$

$\boxed{5(7^{\frac{3}{4}}) \text{ OR } 5\sqrt[4]{7^3}}$

Simplifying Radical Expressions

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

TASK 3: Simplify the radical expression.

a) $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} = \boxed{3\sqrt[3]{5}}$

b) b) $\frac{\sqrt[5]{7}}{\sqrt[5]{8}}$ $\boxed{\sqrt[5]{\frac{7}{8}}}$

c) c) $\sqrt[3]{4} \cdot \sqrt[3]{128} = \sqrt[3]{512} = \boxed{8}$

Rationalizing the Denominator

Reminder about conjugates $a\sqrt{b} + c\sqrt{d}$ & $a\sqrt{b} - c\sqrt{d}$!!!

TASK 4: Rationalize the denominator by multiplying the conjugate.

a) $\frac{1}{5+\sqrt{3}} \frac{(5-\sqrt{3})}{(5-\sqrt{3})} = \boxed{\frac{5-\sqrt{3}}{22}}$

$25-\sqrt{9}$

b) $\frac{1}{\sqrt{7}-2} \frac{(-\sqrt{7}+2)}{(-\sqrt{7}+2)} = \frac{-\sqrt{7}+2}{-\cancel{11}}$

$-\sqrt{49}-4$

c) $\frac{3}{6-\sqrt{2}} \frac{(6+\sqrt{2})}{(6+\sqrt{2})} = \frac{18+3\sqrt{2}}{34}$

$36-\sqrt{4}$

Adding & Subtracting

a) $7\sqrt[5]{12} - 1\sqrt[5]{12}$

$6\sqrt[5]{12}$

b) $4\left(9^{\frac{2}{3}}\right) + 8\left(9^{\frac{2}{3}}\right)$

$12\left(9^{\frac{2}{3}}\right)$

c) $\sqrt[3]{5} + \sqrt[3]{40}$

$\sqrt[3]{5} + 2\sqrt[3]{5} = \boxed{3\sqrt[3]{5}}$

d) $5\sqrt{y} + 6\sqrt{y}$

$11\sqrt{y}$

e) $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

$12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2}$

$\boxed{9z\sqrt[3]{2z^2}}$

f) $\sqrt{9w^5} - w\sqrt[2]{w^3}$

$3w^2\sqrt{w} - w^2\sqrt{w}$

$\boxed{2w^2\sqrt{w}}$

Simplifying Variable Expressions

a) $\sqrt[3]{27q^9} = \sqrt[3]{3 \cdot 3 \cdot 3\,qqqqqqqqq}$

$\boxed{3q^3}$

b) $\sqrt[4]{\frac{x^4}{y^{16}}} = \frac{\cancel{\sqrt[4]{xxxx}}}{\sqrt[4]{yyyyyyyyyyyyyyyy}}$

$\boxed{\frac{x}{y^4}}$

c) $\frac{14xy^{\frac{1}{3}}}{2x^{\frac{3}{4}}z^{-6}} = 7x^{1-\frac{3}{4}}y^{\frac{1}{3}}z^{6}$

$\boxed{7x^{\frac{1}{4}}y^{\frac{1}{3}}z^{6}}$

Still need help with: