5.2 Solve a System of Equations by Substitution

5.2 Lesson

What You Will Learn

▶ Solve systems of linear equations by substitution.
▶ Use systems of linear equations to solve real-life problems.

Core Vocabulary

Previous
system of linear equations
solution of a system of
   linear equations

Solving Linear Systems by Substitution

Another way to solve a system of linear equations is to use substitution.
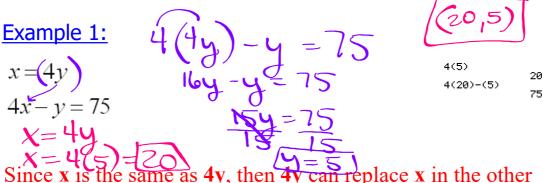
Core Concept

Solving a System of Linear Equations by Substitution

Step 1 Solve one of the equations for one of the variables.

Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3 Substitute the value from Step 2 into one of the original equations and solve.

Nov 9-10:07 AM

Substitution: **An exact method of determining a system of equations by replacing (substituting) a variable in one equation with the algebraic expression in which that variable can be represented by from the other equation.**

Example 1:

$$x = (4y)$$
$$4x - y = 75$$

$$4(4y) - y = 75$$
$$16y - y = 75$$
$$15y = 75$$
$$\frac{15y}{15} = \frac{75}{15}$$
$$y = 5$$

$$x = 4y$$
$$x = 4(5) = 20$$

$$(20, 5)$$

$$4(5)$$
$$4(20) - (5) \qquad \begin{array}{c} 20 \\ 75 \end{array}$$

Since **x** is the same as **4y**, then **4y** can replace **x** in the other equation because you are looking for a common ordered pair (x, y) for both equations.

Dec 18-10:24 AM

Example 1:

Solve:

$$x = 4y$$

$$4x - y = 75$$

Core Concept

Solving a System of Linear Equations by Substitution

Step 1 Solve one of the equations for one of the variables.

Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

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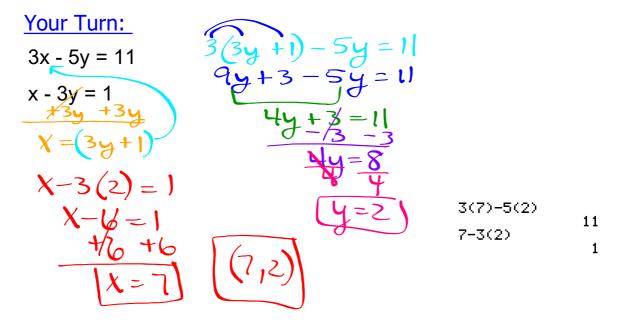
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Solve each system of equations using substitution.

Example 2:

4x + y = 12
-2x - 3y = 14

$-4x \qquad -4x$

$y = (-4x + 12)$

$-2x - 3(-4x + 12) = 14$

$-2x + 12x - 36 = 14$

$10x - 36 = 14$

$ +36 \quad +36$

$\frac{10x}{10} = \frac{50}{10} \quad \boxed{x = 5}$

$4(5) + y = 12$
$20 + y = 12$
$-20 \qquad -20$
$\boxed{y = -8}$

$\boxed{(5, -8)}$

$4(5) + (-8)$

$-2(5) - 3(-8)$

12

14

■

Example 3:

2x + 2y = 8
x + y = -2

$-x \qquad -x$

$y = (-x - 2)$

$2x + 2(-x - 2) = 8$

$2x - 2x - 4 = 8$

$-4 \neq 8 \quad \boxed{\varnothing}$ no solution

Dec 18-2:01 PM

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<u>Your Turn:</u>

$3x - 5y = 11$

$x - 3y = 1$

$\underline{+3y \quad +3y}$

$x = (3y + 1)$

$3(3y + 1) - 5y = 11$

$9y + 3 - 5y = 11$

$4y + 3 = 11$

$\underline{\quad -3 \quad -3}$

$\dfrac{4y}{4} = \dfrac{8}{4}$

$\boxed{y = 2}$

$x - 3(2) = 1$

$x - 6 = 1$

$\underline{+6 \quad +6}$

$\boxed{x = 7}$

$\boxed{(7, 2)}$

$3(7) - 5(2) \qquad 11$

$7 - 3(2) \qquad 1$

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<u>Anytime while solving the equations and you get a final equation of:</u>

$\boxed{\begin{array}{l} -8 = -8 \;\checkmark \\ 5 = 5 \;\checkmark \\ \tfrac{3}{4} = \tfrac{3}{4}\;\checkmark \end{array}}$ The variable drops out so the equations represent the same line and have:

Infinite solutions
Consistent and Dependent ∞ coinciding lines
PR

$\boxed{\begin{array}{l} -8 \neq 5 \;\times \\ 6 \neq -2 \;\times \\ \tfrac{3}{4} \neq 6 \;\times \end{array}}$ Or the equations represent parallel lines and have:

No solutions
Inconsistent $\varnothing$ lines are parallel
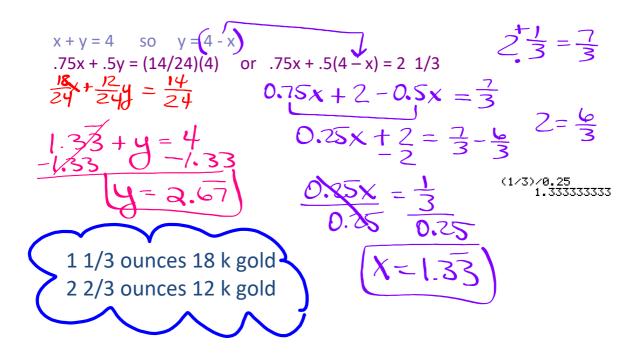
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Example 4:

Gold is alloyed with different metals to make it hard enough to be used in jewelry. The amount of gold present in a gold alloy is measured in 24ths called karats. 24-karat gold is $\frac{24}{24}$ or 100% gold. Similarly, 18-karat gold is $\frac{18}{24}$ or 75% gold. How many ounces of 18-karat gold should be added to an amount of 12-karat gold to make 4 ounces of 14-karat gold?

1 equation for ounces

1 equation for amount or % of gold

x= oz of 18-k gold

y= oz of 12-k gold

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$x + y = 4$ so $y = (4 - x)$

$.75x + .5y = (14/24)(4)$ or $.75x + .5(4 - x) = 2 \ 1/3$

$\frac{18}{24}x + \frac{12}{24}y = \frac{14}{24}$

$0.75x + 2 - 0.5x = \frac{7}{3}$

$0.25x + 2 = \frac{7}{3} - \frac{6}{3}$

$1.33 + y = 4$
$-1.33 \qquad -1.33$

$\boxed{y = 2.67}$

$\frac{0.25x}{0.25} = \frac{\frac{1}{3}}{0.25}$

$2\frac{+1}{3} = \frac{7}{3}$

$2 = \frac{6}{3}$

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(1/3)/0.25
    1.333333333
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$\boxed{x = 1.33}$

> 1 1/3 ounces 18 k gold
> 2 2/3 ounces 12 k gold

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A: 2, 4, 8, 14-20 (e), 26-30(e), 38, 40

B: 1, 2, 4 - 20 (e), 28, 38, 40

C: 4 - 20 (e), 28