5.2 Some Probability Rules Compound Events with work

HW: pg. 198: 3, 5, 7, 11, 17, 19, 21, 23

3. a) 1 b) 0

5. No, the probability was stated for drivers in the age range from 18 to 24. We have no information for other age groups.

7. 627/1010 ≈ 0.62

11. a) MMM, MMF, MFM, MFF, FMM, FMF, FFM, FFF

b) P(MMM) = 1/8. P(at least one female) = 1 - P(MMM) = 7/8

17. a) P(0) = 15/375, P(1) = 71/375, P(2) = 124/375, P(3) = 131/375, P(4) = 34/375

b) Yes.

19. a) P(6a - 12p) = 290/966, P(12p - 6p) = 135/966, P(6 - 12a) = 319/966, P(12a - 6a) = 222/966

b) YES

21. proof b) 2/17 or 0.118 c) 3/8 or 0.375

23. a) 0.46 b) 0.43 c) 0.20 d) 0.57

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5.2 Some Probability Rules - Compound Events

Essential Question:

- What makes an event compound?
- How does conditionals change the probability?

Focus Points:

- Compute probabilities of general compound events.
- Compute probabilities involving independent events or mutually exclusive events.
- Use survey results to compute conditional probabilities.

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Conditional Probability and Multiplication Rules

Two events are independent if the *occurrence or nonoccurrence* of one event does not change the probability that the other event will occur.

MULTIPLICATION RULES

A: coin H: .5
B: 2 1/6

Independent: P(A and B) = P(A)* P(B) = (.5)(1/6)

ANY EVENT: P(A and B) = P(A)*P(B|A)

*** P(B|A) is conditional probability!!!

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Conditional Probability (when P(B) ≠ 0)

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

given that

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Example 1: Two Dice

Suppose you are going to throw two fair dice.
What is the probability of getting a 5 on each die?

Are the events independent? yes

So what formula should we use? $P(A \cap B) = P(A)P(B)$

What is the P(5) = $\frac{1}{6}$

What is the P(5 ∩ 5) = $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \boxed{\frac{1}{36}}$

* we could draw out our sample space too*

11 12 13 14 15 16 ... 55 21 22 23 24 25 26 ... 61 62 63 64 65 66 36

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Example 2: Colored balls

Consider a collection of 6 balls that are identical except in color. There are 3 green balls, 2 purple balls, and 1 red ball.

Compute the probability of drawing 2 green balls from the collection if the first ball is *not replaced* before the second ball is drawn.

Are these events independent? no

What formula do we use? $P(A|B) = \frac{P(A \cap B)}{P(B)}$

What is the P(FG) = $\frac{3}{6} = \frac{1}{2}$

What is the P(SG) = $\frac{2}{5}$

What is the P(FG ∩ SG) = $\left(\frac{1}{2}\right)\left(\frac{2}{5}\right) = \frac{2}{10} = \frac{1}{5}$

Does the sample space agree?

GG GP GR

FG = first green
SG = second green
B = FG

$P(SG|FG) = \frac{P(SG \cap FG)}{P(FG)} = \frac{\frac{1}{5}}{\frac{1}{2}} = \boxed{\frac{2}{5}}$

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5.2 Some Probability Rules Compound Events with work

PROCEDURE

HOW TO USE THE MULTIPLICATION RULES:

1. Determine whether A and B are independent events. If P(A) = P(A|B), then the events are independent.
2. If A and B are independent events, P(A *and* B) = P(A)*P(B)
3. If A and B are any events, P(A *and* B) = P(A)*P(B|A) OR P(A *and* B) = P(B)*P(A|B)

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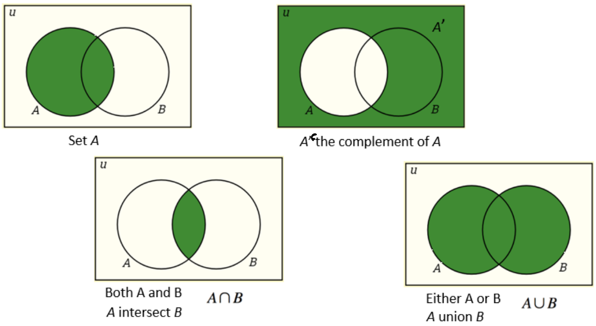

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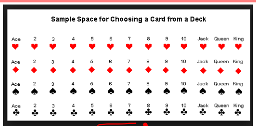
Example 3: Deck of cards

Consider a deck of cards with 52 total cards. The cards include four suits (hearts, diamonds, spades, and clubs). Some cards are red and some are black. The sample space of all cards is shown below.



a) Suppose we select one card at random from the deck. Find the probability that the card is red.

$P(R) = \frac{26}{52} = \boxed{\frac{1}{2}}$

b) Select one card at random from the deck. What is the probability that the ~~student~~ card is either red or a heart?

$P(R \cup H) = \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \boxed{\frac{3}{4}}$

$P(H) = \frac{13}{52} = \frac{1}{4}$

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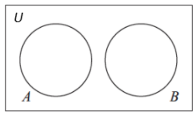
Two events are **mutually exclusive** or **disjoint** if they cannot occur together. In particular, events A and B are **mutually exclusive** if

P(A and B) = 0.



A and B are disjoint sets

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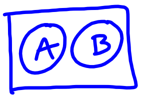
ADDITION RULES

Determine if it is **and** or **or**. If **OR** then addition and multiple ways that it can be solved.

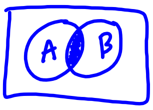
Mutually Exclusive Events A and B

P(A ∪ B) = P(A) + P(B)



Addition for ANY Event A and B

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)



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PROCEDURE

HOW TO USE THE ADDITION RULES:

1. Determine whether A and B are mutually exclusive events. If P(A & B) = 0, then the events are disjoint.
2. If A and B are disjoint events, P(A or B) = P(A) + P(B)
3. If A and B are ANY EVENT, P(A or B) = P(A) + P(B) - P(A and B)

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Example 4: Monopoly

Shelby is playing Monopoly. On her next move she needs to throw a sum bigger than 8 on the two dice in order to land on her own property and pass Go. what is the probability that ~~Laura~~ Shelby will roll a sum bigger than 8?

Is this an and or an or? S: {9, 10, 11, 12}

Are the sums larger than 8 mutually exclusive? yes

What is the ...

63 36 | 54 45 | 55 64 46 | 65 56 | 66

P(9) $\frac{4}{36} = \frac{1}{9}$ P(10) $\frac{3}{36} = \frac{1}{12}$ P(11) $\frac{2}{36} = \frac{1}{18}$ P(12) $= \frac{1}{36}$

Final P(>8) =

$$\frac{1}{9} + \frac{1}{12} + \frac{1}{18} + \frac{1}{36} = \frac{10}{36} = \boxed{\frac{5}{18}}$$

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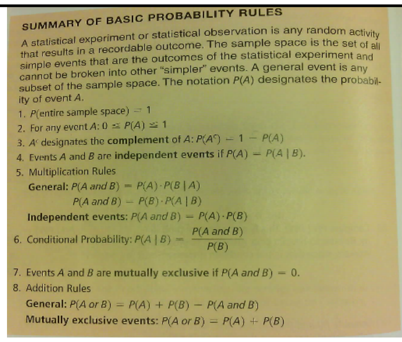

SUMMARY OF BASIC PROBABILITY RULES

A statistical experiment or statistical observation is any random activity that results in a recordable outcome. The sample space is the set of all simple events that are the outcomes of the statistical experiment and cannot be broken into other "simpler" events. A general event is any subset of the sample space. The notation $P(A)$ designates the probability of event A.

1. P(entire sample space) = 1
2. For any event A: $0 \le P(A) \le 1$
3. A^c designates the **complement** of A: $P(A^c) = 1 - P(A)$
4. Events A and B are **independent events** if $P(A) = P(A \mid B)$.
5. Multiplication Rules
   General: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
   $P(A \text{ and } B) = P(B) \cdot P(A \mid B)$
   Independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
6. Conditional Probability: $P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$
7. Events A and B are **mutually exclusive** if $P(A \text{ and } B) = 0$.
8. Addition Rules
   General: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
   Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$

p. 214

Nov 1-9:32 AM

HW: p. 215: 1 - 25 odd

1. No. By definition, disjoint events cannot occur together.

3. a) 0.7 b) 0.6

5. a) 0.08 b) 0.04

7. a) 0.15 b) 0.55

9. P(A∩B) is the probability that both events A and B occur. It cannot exceed the probability that either event occurs. When the assigned probabilities are used to get P(A|B), the result exceeds 1.

11. a) Bc the events are mutually exclusive, A cannot occur if B occurred. P(A|B) = 0.
    b) Bc P(A|B) ≠ P(A), the events A and B are not independent.

13. a) P(A and B). b) P(B|A). c) P(A^c|B). d) P(A or B) e) P(B^c or A).

15. a) 0.2; yes. b) 0.4; yes. c) 0.8

17. a) yes b) 0.028 c) 0.028 d) 0.056

19. a) $\frac{5}{36}$ b) $\frac{1}{12}$ c) $\frac{2}{9}$, yes

21. a) No b) $\frac{4}{663}$ c) $\frac{4}{663}$ d) $\frac{8}{663}$

23. a) yes b) $\frac{1}{169}$ c) $\frac{1}{169}$ d) $\frac{2}{169}$

25. a) 0.63 b) 0.78 c) 0.41 d) 0.49

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