HW: p. 215: 1 - 25 odd

1. No. By definition, disjoint events cannot occur together.

3. a) 0.7 b) 0.6

5. a) 0.08 b) 0.04

7. a) 0.15 b) 0.55

9. P(A∩B) is the probability that both events A and B occur. It cannot exceed the probability that either event occurs. When the assigned probabilities are used to get P(A|B), the result exceeds 1.

11. a) Bc the events are mutually exclusive, A cannot occur if B occurred. P(A|B) = 0.
    b) Bc P(A|B) ≠ P(A), the events A and B are not independent.

13. a) P(A and B). b) P(B|A). c) P(A^c|B). d) P(A or B) e) P(B^c or A).

15. a) 0.2; yes. b) 0.4; yes. c) 0.8

17. a) yes b) 0.028 c) 0.028 d) 0.056

19. a) $\frac{5}{36}$ b) $\frac{1}{12}$ c) $\frac{2}{9}$, *yes*

21. a) No b) $\frac{4}{663}$ c) $\frac{4}{663}$ d) $\frac{8}{663}$

23. a) yes b) $\frac{1}{169}$ c) $\frac{1}{169}$ d) $\frac{2}{169}$

25. a) 0.63 b) 0.78 c) 0.41 d) 0.49

Nov 1-9:35 AM

5.3 Trees and Counting Techniques

Essential Questions:

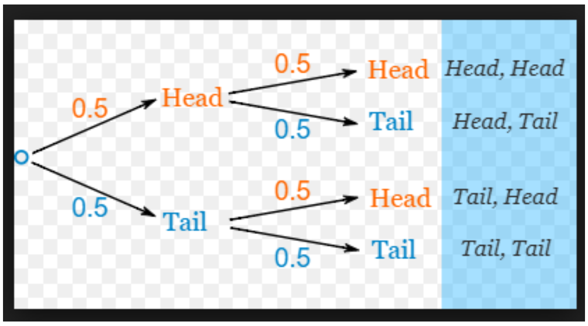
Which method is the best for creating a sample space?

Focus Points:

- Organize outcomes in a sample space using tree diagrams.
- Compute number of ordered arrangements of outcomes using permutations.
- Compute number of (nonordered) groupings of outcomes using combinations.
- Explain how counting techniques relate to probability in everyday life.

Nov 2-1:57 PM

A tree diagram gives a visual display of the total number of outcomes of an experiment consisting of a series of events.



** tree diagrams are a good way to visualize a sample space involving several stages. **

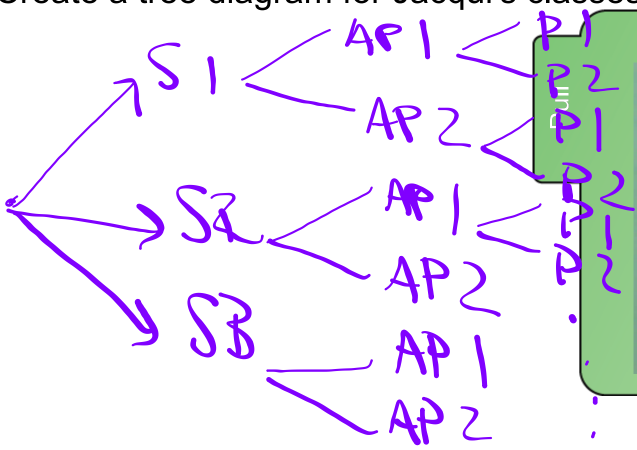
Nov 2-2:01 PM

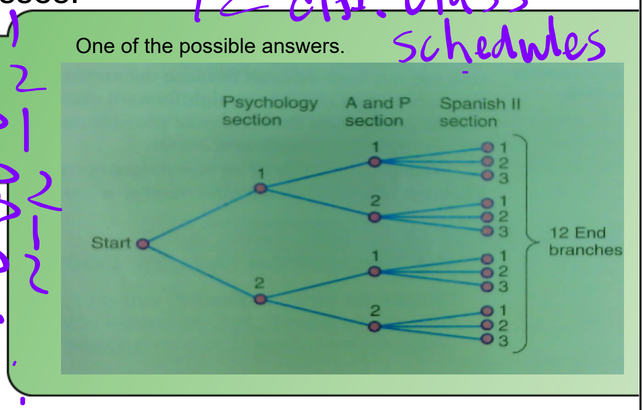
Example 1: Jacqui's class schedule

Jacqueline is in a nursing program and is required to take a course in psychology and one in physiology (A and P) next semester. She also wants to take Spanish II. If there are two sections of psychology, two of A and P, and three of Spanish II, how many different class schedules can Jacqui choose from? (assume that the times of the classes do not conflict.)

Create a tree diagram for Jacqui's classes.

12 diff. class schedules





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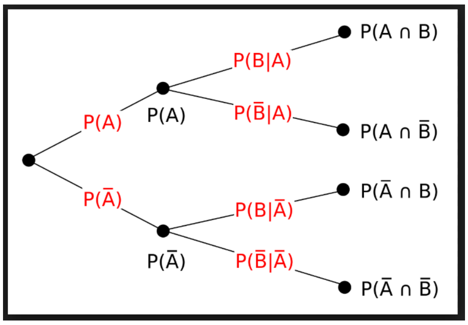
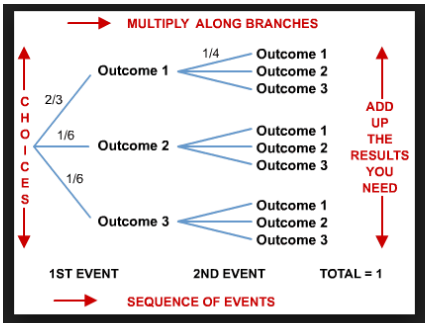
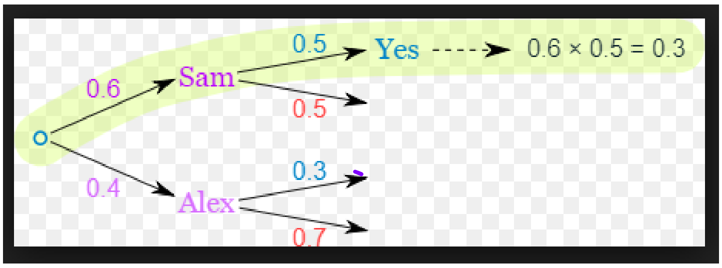




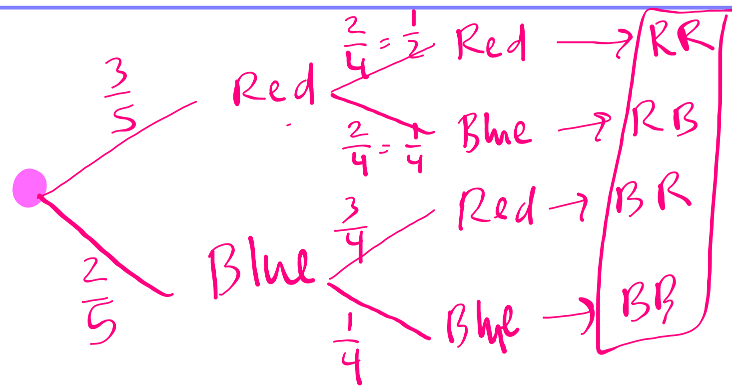
Several ways to use tree diagrams: determine probability



Nov 2-2:03 PM

Example 2: Balls in an Urn

Suppose there are five balls in an urn. They are identical except for color. Three of the balls are red and two are blue. You are instructed to draw out one ball, note its color, and set it side. Then you are to draw out another ball and note its color. What are the outcomes of the experiment? What is the probability of each outcome?



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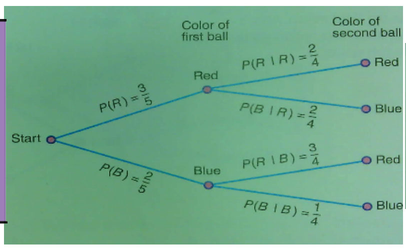


$$P(R \text{ on 1st and } R \text{ on 2nd}) = P(R)\cdot P(R \mid R) = \frac{3}{5}\cdot\frac{2}{4} = \frac{3}{10}$$
$$P(R \text{ on 1st and } B \text{ on 2nd}) = P(R)\cdot P(B \mid R) = \frac{3}{5}\cdot\frac{2}{4} = \frac{3}{10}$$
$$P(B \text{ on 1st and } R \text{ on 2nd}) = P(B)\cdot P(R \mid B) = \frac{2}{5}\cdot\frac{3}{4} = \frac{3}{10}$$
$$P(B \text{ on 1st and } B \text{ on 2nd}) = P(B)\cdot P(B \mid B) = \frac{2}{5}\cdot\frac{1}{4} = \frac{1}{10}$$

Nov 2-2:13 PM

Thanksgiving is coming up!!!

How many different ways can eight chairs be filled with eight people?

8 · 7 · 6 · 5 · 4 · 3 · 2 · 1 = ?

! is read "factorial"

8! is read "8 factorial"

8! = 8*7*6*5*4*3*2*1 = 40,320

Nov 2-2:25 PM

Factorial Notation

$0! = 1$

$1! = 1$

$n! = n(n - 1)(n - 2)...1$

$n=8 \quad 8! = 8(8-1)(8-2)\cdots 1$

$8(7)(6)\cdots\cdots(1) = 40{,}320$

Nov 2-2:30 PM

Now, we only have 4 chairs for dinner, but 8 people show up. Now what do we do?

Nov 2-2:31 PM

Permutation formula to compute 4 chairs but 8 people. The permutations rule is really another version of the *multiplication rule*.

Counting Rule for PERMUTATIONS:

The number of ways to *arrange in order n* distinct objects, taking them r at a time, is

$$P_{n,r} = \frac{(n!)}{((n-r)!)}$$

n= 8
r= 4

where n and r are whole numbers and n ≥ r. Another commonly used notation for permutations is nPr.

Nov 2-2:33 PM

Example 3: Chairs < People

Four chairs eight people. Compute the number of possible ordered seating arrangements for eight people and four chairs.

What formula? $P_{n,r} = \frac{n!}{(n-r)!} = P_{8,4} = \frac{8!}{(8-4)!}$

n = 8

r = 4

```
³√4
        1.587401052
(8!)/(8-4)!
               1680
8 nPr 4
               1680
```

$= \frac{8\cdot7\cdot6\cdots\cancel{4}}{\cancel{4\cdot3\cdot2\cdot1}}$

$= 8\cdot7\cdot6\cdot5$

$\boxed{P_{8,4} = 1680}$

Nov 6-8:12 AM

In permutations, the ***order*** of occurrence is important, while in combinations it is not even taken into consideration.

COUNTING RULE FOR COMBINATIONS

the number of combinations of *n* objects take *r* at a time is

$$C_{n,r} = \frac{(n!)}{(r!(n-r)!)}$$

where *n* and *r* are whole numbers and n ≥ r. Other commonly used notations for combinations include nCr and $\binom{n}{r}$.

Nov 6-8:15 AM

Example 4: Political Science Class

In your political science class, you are assigned to read any 4 books from a list of 10 books. How many different groups of 4 are available from the list of 10?

What formula? $C_{n,r} = \frac{n!}{r!(n-r)!}$

n = 10

r = 4

$C_{10,4} = \frac{10!}{4!(10-4)!} = \frac{10!}{4!(6)!} = \boxed{210}$

What are the other 2 notations we should know?

${}_{10}C_4$ OR $C\binom{10}{4}$

```
(10!)/(4!(10-4)▸
              210
10 nCr 4
              210
```

Nov 6-8:20 AM

HOW TO DETERMINE THE NUMBER OF OUTCOMES OF AN EXPERIMENT

1) If the experiment consists of a series of stages with various outcomes, use the multiplication rule or tree diagram.

2) If the outcomes consist of ordered subgroups of *r* items taken from a group of *n* items, use the permutations rule, $P_{n,r}$.

$$P_{n,r} = \frac{n!}{(n-r)!}$$

3) If the outcomes consist of nonordered subgroups of *r* items taken from a group of *n* items, use the combinations rule, $C_{n,r}$.

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

Nov 6-8:23 AM

5.3 Trees and Counting Techniques with work

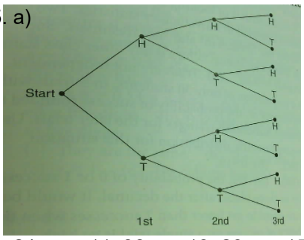
HW: pg. 229: 1 - 27 odd

1. The permutations rule counts the number of different arrangements of r items out of n distinct items, whereas the combinations rule counts only the number of groups of r items out of n distinct items. The number of permutations is larger that the number of combinations.

3. a) Use the combinations rule, since only the items in the group and not their arrangement is of concern.

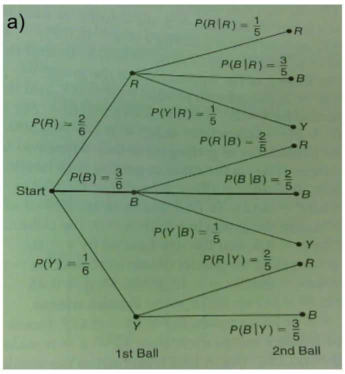
   b) Use the permutations rule, since the number of arrangements within each group is of interest.

5. a)



b) 3 c) 3/8

7. a)



b) RR, 1/15, RB, 1/5, RY, 1/15, BR, 1/5; BB, 1/5; BY, 1/10; YR, 1/15; YB, 1/10

9. 24 11. 36 13. 20 15. 5040 17. 10 19. 1

21. $P_{15,3}$ = 2.730 23. $P_{5,3}$ = 60 25. $C_{15,5}$ = 3003

27. a) $C_{12,5}$ = 792 b) $C_{7,6}$ = 7 c) 0.008

Nov 6-8:26 AM