5.4 Solving Special Systems

Essential Question Can a system of linear equations have no solution or infinitely many solutions?

What You Will Learn

- Determine the numbers of solutions of linear systems.
- Use linear systems to solve real-life problems.

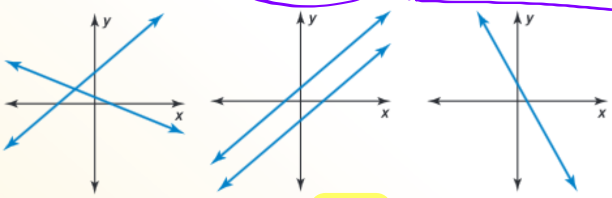
The Numbers of Solutions of Linear Systems

Core Concept

Solutions of Systems of Linear Equations

A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.

One solution	No solution	Infinitely many solutions



coinciding

The lines intersect.	The lines are parallel.	The lines are the same.
	$m=m$, $b \neq b$	$m=m$, $b=b$

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EXAMPLE 1 Solving a System: No Solution

Solve the system of linear equations.

$y = 2x + 1$ Equation 1 $m=2$ $b=1$

$y = 2x - 5$ Equation 2 $m=2$ $b=-5$

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ANOTHER WAY

You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$-2x + y = 1$

$-2x + y = -5.$

This system has no solution because $-2x + y$ cannot be equal to both 1 and −5.

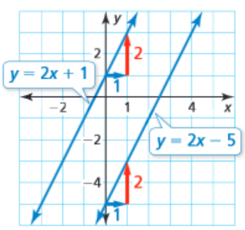
SOLUTION

Method 1 Solve by graphing.

Graph each equation.

The lines have the same slope and different y-intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.



- So, the system of linear equations has no solution.

Method 2 Solve by substitution.

Substitute $2x - 5$ for y in Equation 1.

$y = 2x + 1$ Equation 1

$2x - 5 = 2x + 1$ Substitute $2x - 5$ for y.

$-5 = 1$ ✗ Subtract $2x$ from each side.

- The equation $-5 = 1$ is never true. So, the system of linear equations has no solution.

STUDY TIP

A linear system with no solution is called an *inconsistent system.*

$-5 \neq 1$ X $\Rightarrow \boxed{\emptyset}$

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EXAMPLE 2 Solving a System: Infinitely Many Solutions

Solve the system of linear equations.

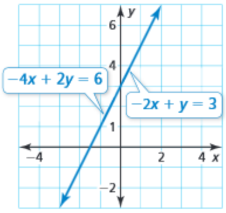
$-2x + y = 3$ Equation 1

$-4x + 2y = 6$ Equation 2

SOLUTION

Method 1 Solve by graphing.

Graph each equation.



The lines have the same slope and the same y-intercept. So, the lines are the same. Because the lines are the same, all points on the line are solutions of both equations.

▶ So, the system of linear equations has infinitely many solutions.

Coinciding

$\boxed{\text{inf. many sol.}}$

Method 2 Solve by elimination.

Step 1 Multiply Equation 1 by −2.

$-2x + y = 3$ → Multiply by −2. → $4x - 2y = -6$ Revised Equation 1

$-4x + 2y = 6$ → $-4x + 2y = 6$ Equation 2

Step 2 Add the equations.

$4x - 2y = -6$ Revised Equation 1

$-4x + 2y = 6$ Equation 2

$0 = 0$ Add the equations.

▶ The equation $0 = 0$ is always true. So, the solutions are all the points on the line $-2x + y = 3$. The system of linear equations has infinitely many solutions.

$\cancel{-2x} + y = 3$
$\cancel{+2x} \quad +2x$
$y = 2x + 3 \quad m = 2 \quad b = 3$

$\cancel{-4x} + 2y = 6$
$\cancel{+4x} \quad +4x$
$\frac{\cancel{2}y}{\cancel{2}} = \frac{4x}{2} + \frac{6}{2}$
$y = 2x + 3 \quad m = 2 \quad b = 3$

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YOUR TURN:

Solve the system of linear equations.

$y = 2x + 1 \quad m = 2 \quad b = 1$

$y = (2x - 5) \quad m = 2 \quad b = -5$

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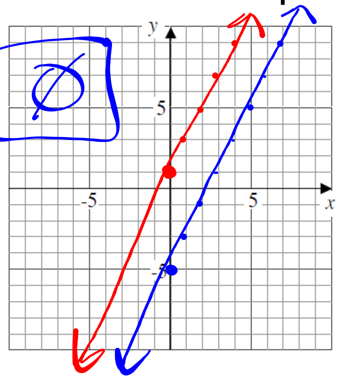
Method 1: Graphing



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Method 2: Algebraically sub elim.

$\cancel{2x} - 5 = \cancel{2x} + 1$
$\cancel{-2x} \quad \cancel{-2x}$
$-5 \neq 1$ X

$\boxed{\varnothing}$

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A: 8, 16, 22, 24 - 27, 32, 34

B: 1, 2, 8, 14, - 24(e), 25 - 27, 34

C: 6 - 22 (e), 26, 33