5.4 Special Products

OBJECTIVE 1: Using the FOIL Method

The FOIL Method

F stands for the product of the **First** terms. $(3x + 1)(2x + 5)$

$(3x)(2x) = 6x^2$ **F**

O stands for the product of the **Outer** terms. $(3x + 1)(2x + 5)$

$(3x)(5) = 15x$ **O**

I stands for the product of the **Inner** terms. $(3x + 1)(2x + 5)$

$(1)(2x) = 2x$ **I**

L stands for the product of the **Last** terms. $(3x + 1)(2x + 5)$

$(1)(5) = 5$ **L**

Example 1: Multiply $(x - 3)(x + 4)$ using FOIL.

$x^2 + 4x - 3x - 12$

$x^2 + x - 12$

Practice 1: $(x + 2)(x - 5)$.

$x^2 - 5x + 2x - 10$

$x^2 - 3x - 10$

Example 2: Multiply (5x - 7)(x - 2) by FOILing.

$5x^2 - 10x - 7x + 14$

$\boxed{5x^2 - 17x + 14}$

Practice 2: (4x - 9)(x - 1)

$4x^2 - 4x - 9x + 9$

$\boxed{4x^2 - 13x + 9}$

Example 3: Multiply: 2(y + 6)(2y - 1).

$2[2y^2 - y + 12y - 6]$

$2[2y^2 + 11y - 6]$

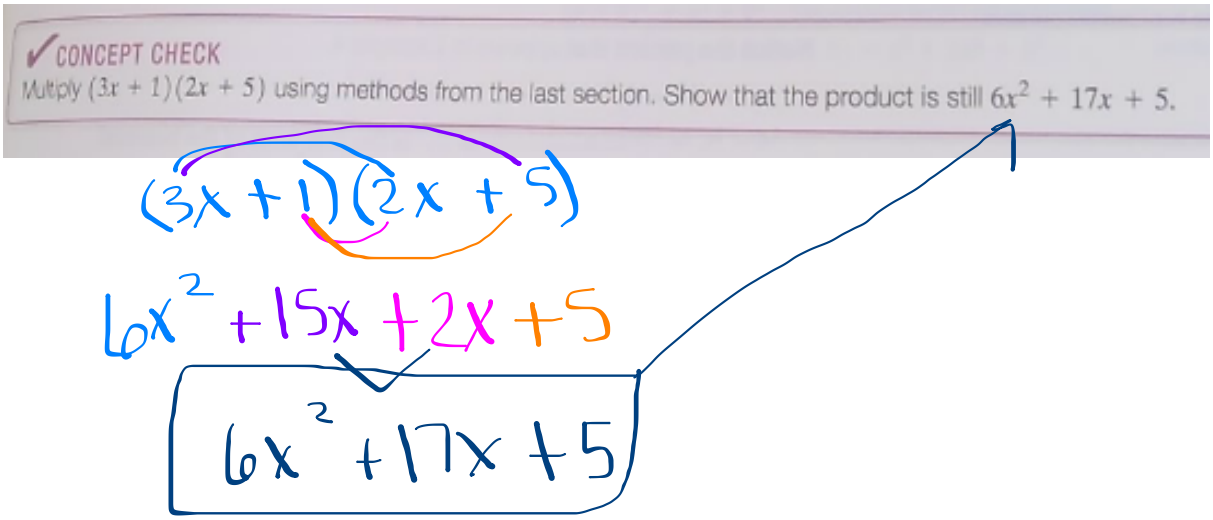
$\boxed{4y^2 + 22y - 12}$

Practice 3: 3(x + 5)(3x - 1)

$3[3x^2 - x + 15x - 5]$

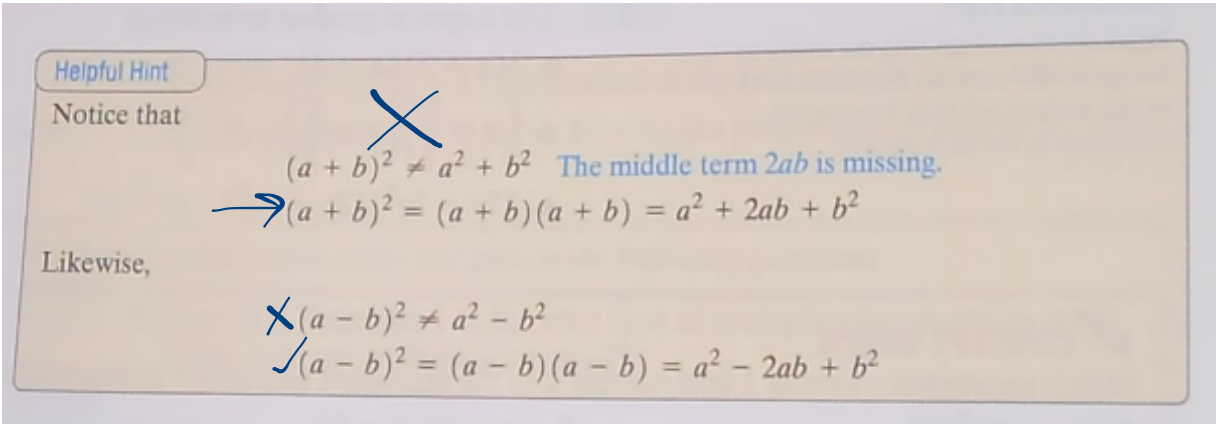
$3[3x^2 + 14x - 5]$

$\boxed{9x^2 + 42x - 15}$



OBJECTIVE 2: Squaring Binomials

Most common mistake is people distribute the square instead of making two sets of parentheses and FOILing.



Example 4: Multiply: $(3y + 1)^2$

$(3y+1)(3y+1)$

$9y^2+3y+3y+1$

$\boxed{9y^2+6y+1}$

Practice 4: $(4x - 1)^2$

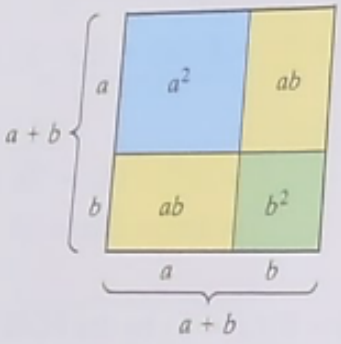
$(4x-1)(4x-1)$

$16x^2-4x-4x+1$

$\boxed{16x^2-8x+1}$

This pattern leads to the following special product that can be used only when squaring a binomial.

This product can be visualized geometrically.



The area of the large square is side · side.

$$\text{Area} = (a + b)(a + b) = (a + b)^2$$

The area of the large square is also the sum of the areas of the smaller rectangles.

$$\text{Area} = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

Thus, $(a + b)^2 = a^2 + 2ab + b^2$.

Squaring a Binomial

A binomial squared is equal to the square of the first term plus or minus twice the product of both terms plus the square of the second term.

$$+\ (a+b)^2 = a^2 + 2ab + b^2$$
$$-\ (a-b)^2 = a^2 - 2ab + b^2$$

Example 5: Use a special product to square each binomial.

a) $(t + 2)^2$

$a = t \quad b = 2$

$t^2 + 2(t)(2) + 2^2$

$t^2 + 4t + 4$

b) $(p - q)^2$

$a = p \quad b = q$

$p^2 - 2pq + q^2$

c) $(2x + 5)^2$

$a = 2x \quad b = 5$

$(2x)^2 + 2(2x)(5) + 5^2$

$4x^2 + 20x + 25$

d) $(x^2 - 7y)^2$

$a = x^2 \quad b = 7y$

$(x^2)^2 - 2(x^2)(7y) + (7y)^2$

$x^4 - 14x^2y + 49y^2$

Practice 5: Use a special product to square each binomial.

a) $(b + 3)^2$

$a = b \quad b = 3$

$(b)^2 + 2(b)(3) + 3^2$

$b^2 + 6b + 9$

b) $(x - y)^2$

$a = x \quad b = y$

$(x)^2 - 2xy + y^2$

c) $(3y + 2)^2$

$a = 3y \quad b = 2$

$(3y)^2 + 2(3y)(2) + 2^2$

$9y^2 + 12y + 4$

d) $(a^2 - 5b)^2$

$a = a^2 \quad b = 5b$

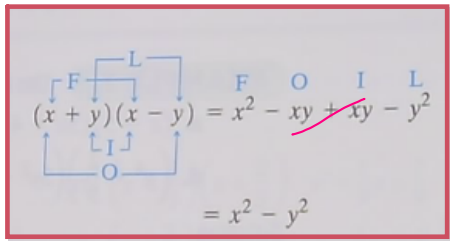
$(a^2)^2 - 2(a^2)(5b) + (5b)^2$

$a^4 - 10a^2b + 25b^2$

OBJECTIVE 3: Multiplying the Sum and Difference of Two Terms

Another special project is the product of the sum and difference of the same two terms, such as (x + y)(x - y). Finding this product by the FOIL method you can see the pattern.

Notice that the middle two terms subtract out. This is because the Outer product is the opposite of the Inner product. Only the difference of squares remains.



Multiplying the Sum and Difference of Two Terms

The product of the sum and difference of two terms is the square of the first term minus the square of the second term.

$$(a + b)(a - b) = a^2 - b^2$$

Example 6: Use a special product to multiply.

a) 4(x + 4)(x - 4)

$4(x^2 - 16)$

$4x^2 - 64$

b) (6t + 7)(6t - 7)

$36t^2 - 49$

c) $\left(x - \frac{1}{4}\right)\left(x + \frac{1}{4}\right)$

$x^2 - \frac{1}{16}$

d) (2q - p)(2q + p)

$4q^2 - p^2$

e) $(3x^2 - 5y)(3x^2 + 5y)$

$9x^4 - 25y^2$

Practice 6: Use a special product to multiply.

a) 3(x + 5)(x - 5)

$3(x^2-25)$

$3x^2-75$

b) (4b - 3)(4b + 3)

$16b^2-9$

c) $\left(x+\frac{2}{3}\right)\left(x-\frac{2}{3}\right)$

$x^2-\frac{4}{9}$

d) (5s - t)(5s + t)

$25s^2-t^2$

e) $(2y - 3z^2)(2y + 3z^2)$

$4y^2-9z^4$

✔ CONCEPT CHECK

Match expression number 1 and number 2 to the equivalent expression or expressions in the list below.

1. $(a+b)^2$ ~~A~~ E

2. $(a+b)(a-b)$ B

A. $(a+b)(a+b)$ B. a^2-b^2 ~~C. a^2+b^2~~ ~~D~~ $a^2-2ab+b^2$ E. $a^2+2ab+b^2$

OBJECTIVE 4: Using Special Products

These are all mixed up and you need to use what you know to multiply the polynomials.

Example 7: Use a special product to multiply, if possible.

a) $(x - 5)(3x + 4)$

$3x^2 + 4x - 15x - 20$

$3x^2 - 11x - 20$

b) $(7x + 4)^2$

$(7x+4)(7x+4)$

$49x^2 + 2(7x)(4) + 16$

$49x^2 + 56x + 16$

c) $(y - 0.6)(y + 0.6)$

$y^2 - 0.36$

d) $\left(y^4 + \frac{2}{5}\right)\left(3y^2 - \frac{1}{5}\right)$

$3y^6 - \frac{1}{5}y^4 + \frac{6}{5}y^2 - \frac{2}{25}$

e) $(a - 3)(a^2 + 2a - 1)$

$a^3 + 2a^2 - a$

$-3a^2 - 6a + 3$

$a^3 - a^2 - 7a + 3$

Practice 7: Use a special product to multiply, if possible.

a) $(4x + 3)(x - 6)$

$4x^2 - 24x + 3x - 18$

$4x^2 - 21x - 18$

b) $(7b - 2)^2$

$(7b-2)(7b-2)$

$49b^2 - 2(7b)(2) + 4$

$49b^2 + 28b + 4$

c) $(y - 0.4)(y + 0.4)$

$y^2 - 0.16$

d) $\left(x^2 - \frac{3}{7}\right)\left(3x^4 + \frac{2}{7}\right)$

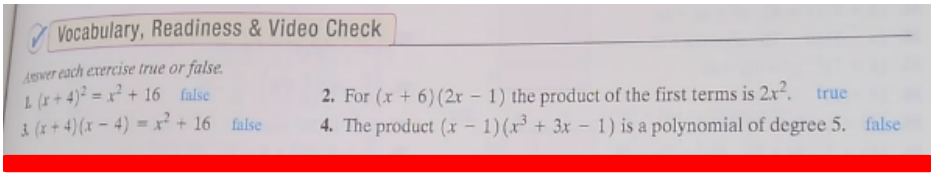
$3x^6 - \frac{9}{7}x^4 + \frac{2}{7}x^2 - \frac{6}{49}$

e) $(x + 1)(x^2 + 5x - 2)$

$x^3 + 5x^2 - 2x$

$x^2 + 5x - 2$

$x^3 + 6x^2 + 3x - 2$



Vocabulary, Readiness & Video Check

Answer each exercise true or false.

1. $(x+4)^2 = x^2 + 16$ false
2. For $(x+6)(2x-1)$ the product of the first terms is $2x^2$. true
3. $(x+4)(x-4) = x^2 + 16$ false
4. The product $(x-1)(x^3+3x-1)$ is a polynomial of degree 5. false

5.4 HW: p. 345

1 - 29 (o), 31 - 77 (eoo), 79 - 93(o)

eoo = every other odd