5.7 Synthetic Division & the Remainder Theorem

OBJECTIVE 1: Using Synthetic Division

This is a shorter form of division, if it is a linear binomial or monomial. It removes the variables and only uses the coefficients. We use a slightly different form and it is almost all the opposite process.

$$\begin{array}{r} 2x^2 + 5x + 2 \\ x - 3\overline{)2x^3 - x^2 - 13x + 1} \\ \underline{2x^3 - 6x^2} \\ 5x^2 - 13x \\ \underline{5x^2 - 15x} \\ 2x + 1 \\ \underline{2x - 6} \\ 7 \end{array} \qquad \begin{array}{r} 2 \quad 5 \quad 2 \\ 1 - 3\overline{)2 - 1 - 13 + 1} \\ \underline{2 - 6} \\ 5 - 13 \\ \underline{5 - 15} \\ 2 + 1 \\ \underline{2 - 6} \\ 7 \end{array}$$

Example 1: Use synthetic division to divide

$$2x^3 - x^2 - 13x + 1 \text{ by } x - 3.$$

$x =$

$x - 3 = 0$
$+3 \quad +3$
$\boxed{x = 3}$

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ + & \downarrow & 6 & 15 & 6 \\ \hline & 2 & 5 & 2 & \boxed{7} \end{array}$$

$\boxed{2x^2 + 5x + 2 \text{ R.} 7}$

Practice 1: $4x^3 - 3x^2 + 6x + 5$ by $x - 1$

$x - 1 = 0$
$x = 1$

$$\begin{array}{r|rrrr} 1 & 4 & -3 & 6 & 5 \\ + & \downarrow & 4 & 1 & 7 \\ \hline & 4 & 1 & 7 & \boxed{12} \end{array}$$

$\boxed{4x^2 + x + 7 \text{ R.} 12}$

Example 2: Use synthetic division to divide

$$x^4 - 2x^3 + 6x + 5 \text{ by } x - 1.$$

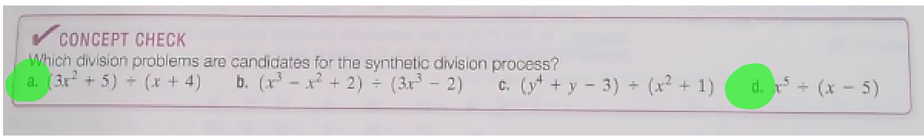
$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 0 & 6 & 5 \\ + & \downarrow & 1 & -1 & -1 & 5 \\ \hline & 1 & -1 & -1 & 5 & \boxed{10} \end{array}$$

$$\boxed{x^3 - x^2 - x + 5 \text{ R. } 10}$$

Practice 2: $x^4 + 3x^3 - 5x^2 + 6x + 12$ by $x + 3$

$$\begin{array}{r|rrrrr} -3 & 1 & 3 & -5 & 6 & 12 \\ + & \downarrow & -3 & 0 & 15 & -30 \\ \hline & 1 & 0 & -5 & 21 & \boxed{-24} \end{array}$$

$$\boxed{x^3 + 0x^2 - 5x + 21 - \frac{24}{x+3}}$$



CONCEPT CHECK

Which division problems are candidates for the synthetic division process?

a. $(3x^2 + 5) \div (x + 4)$ b. $(x^3 - x^2 + 2) \div (3x^3 - 2)$ c. $(y^4 + y - 3) \div (x^2 + 1)$ d. $x^5 \div (x - 5)$

Divisor MUST be a LINEAR binomial or monomial

Helpful Hint
Before dividing by synthetic division, write the dividend in descending order of variable exponents. Any "missing powers" of the variable should be represented by 0 times the variable raised to the missing power.

Example 3: If $P(x) = 2x^3 - 4x^2 + 5$.

$2(2)^3-4(2)^2+5$

5

a) Find P(2) by substitution. $x=2$

$P(2) = 2(2)^3 - 4(2)^2 + 5$ $\boxed{P(2)=5}$ $\boxed{(2,5)}$

b) Use synthetic division to find the remainder when P(x) is divided by x - 2. $x=2$

$$\begin{array}{r|rrrr} 2 & 2 & -4 & 0 & 5 \\ + & \downarrow & 4 & 0 & 0 \\ \hline & 2 & 0 & 0 & \boxed{5} \end{array}$$

Practice 3: If $P(x) = x^3 - 5x - 2$.

$(2)^3-5(2)-2$

-4

a) Find P(2) by substitution. $x=2$

$P(2) = (2)^3 - 5(2) - 2$ $\boxed{P(2) = -4}$ $\boxed{(2,-4)}$

b) Use synthetic division to find the remainder when P(x) is divided by x - 2.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & -2 \\ + & \downarrow & 2 & 4 & -2 \\ \hline & 1 & 2 & -1 & \boxed{-4} \end{array}$$

$(2,-4)$

R. -4

OBJECTIVE 2: Using the Remainder Theorem

Notice that P(2) = 5 and the remainder when P(x) is divided by x - 2 is 5. This is no accident. This is an example of the remainder theorem.

Example 4: Use the remainder theorem AND synthetic division to find P(4) if $P(x) = 4x^6 - 25x^5 + 35x^4 + 17x^2$.

$4(4)^6-25(4)^5+35($
16

$P(4) = 4(4)^6 - 25(4)^5 + 35(4)^4 + 17(4)^2$

$$\begin{array}{r|rrrrrrr} 4 & 4 & -25 & 35 & 0 & 17 & 0 & 0 \\ + & \downarrow & 16 & -36 & -4 & -16 & 4 & 16 \\ \hline & 4 & -9 & -1 & -4 & 1 & 4 & \boxed{16} \end{array}$$

Practice 4: Use the remainder theorem AND synthetic division to find P(3) if $P(x) = 2x^5 - 18x^4 + 90x^2 + 59x$.

$2(3)^5-18(3)^4+90($
15

$P(3) = 2(3)^5 - 18(3)^4 + 90(3)^2 + 59(3)$

$$\begin{array}{r|rrrrrr} 3 & 2 & -18 & 0 & 90 & 59 & 0 \\ + & \downarrow & 6 & -36 & -108 & -54 & 15 \\ \hline & 2 & -12 & -36 & -18 & 5 & \boxed{15} \end{array}$$

5.7 HW

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3, 7, 9 - 35 (o), 39 - 57 (o), 63, 65, 67