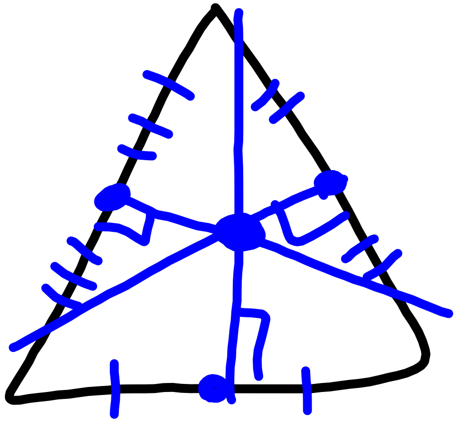
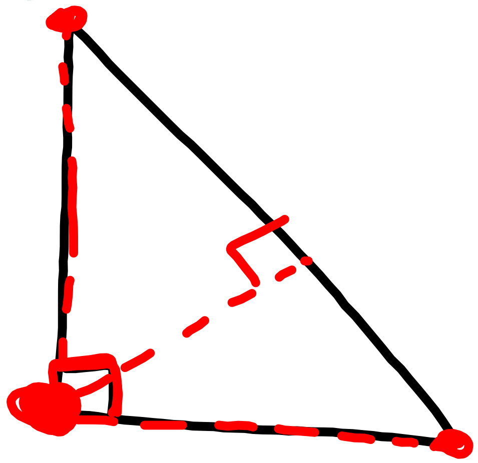
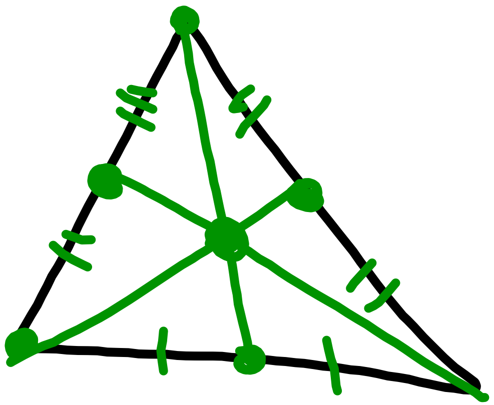
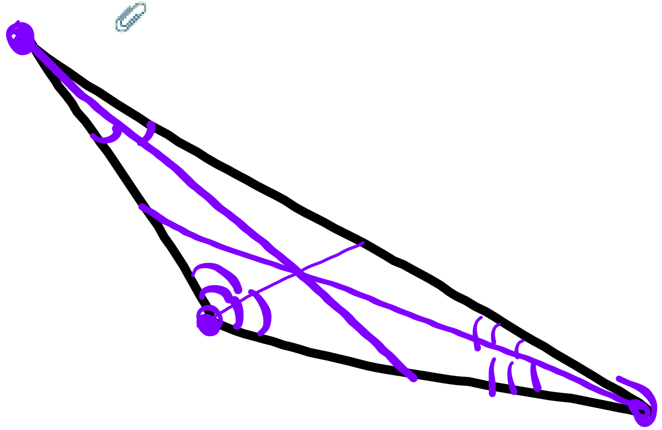
6.1 - 6.3 Special Segments of Triangles MASTERS with work

Perpendicular Bisector	Altitude
perpendicular bisectors.gsp 	Altitudes copy.gsp 
Median	**Angle Bisector**
medians.gsp 	Angle bisectors.gsp 

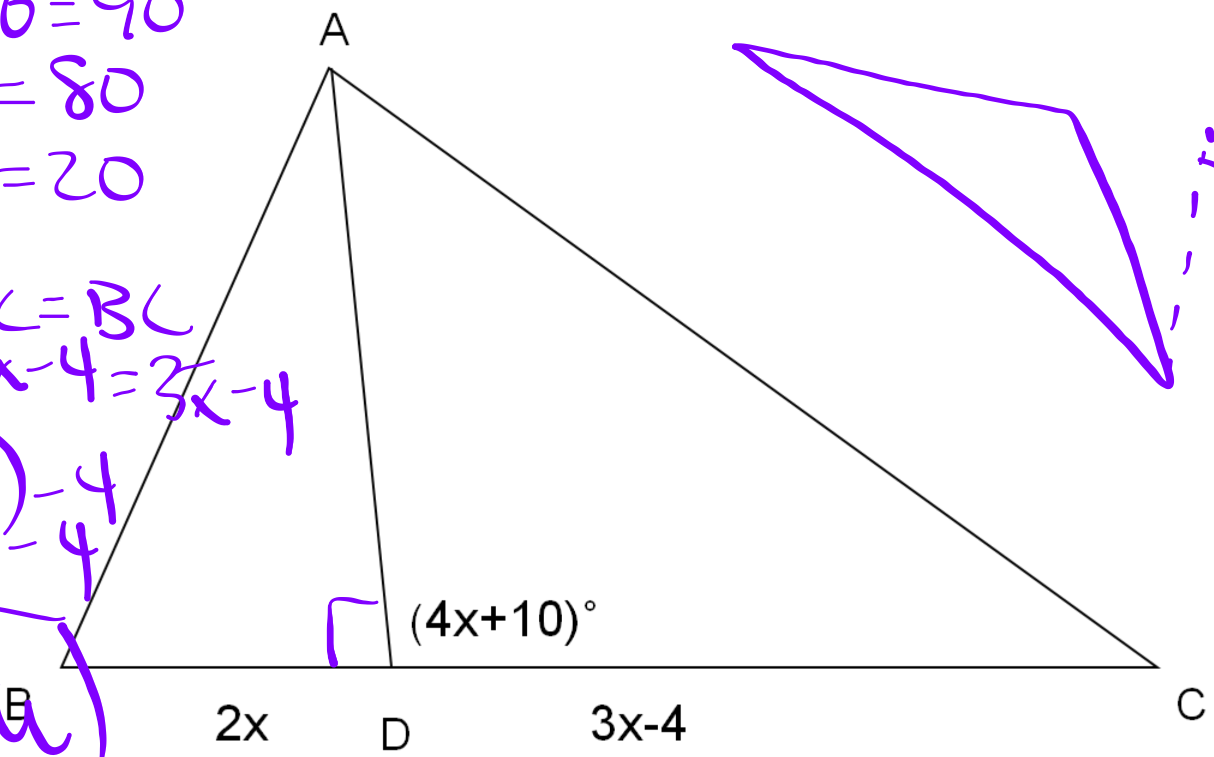
Applying and Solving

- Find BC if $\overline{AD}$ is an altitude of $\triangle ABC$.

$4x+10=90$

$4x=80$

$x=20$



$BD+DC=BC$

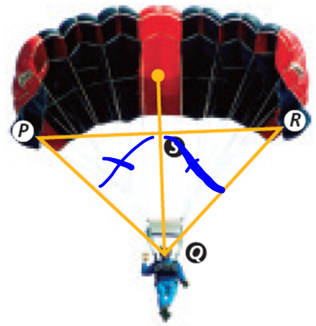
$2x+3x-4=5x-4$

$5(20)-4$

$100-4$

$\boxed{96u}$

S **is equidistant from each pair of suspension lines. What can you conclude about *QS*?**



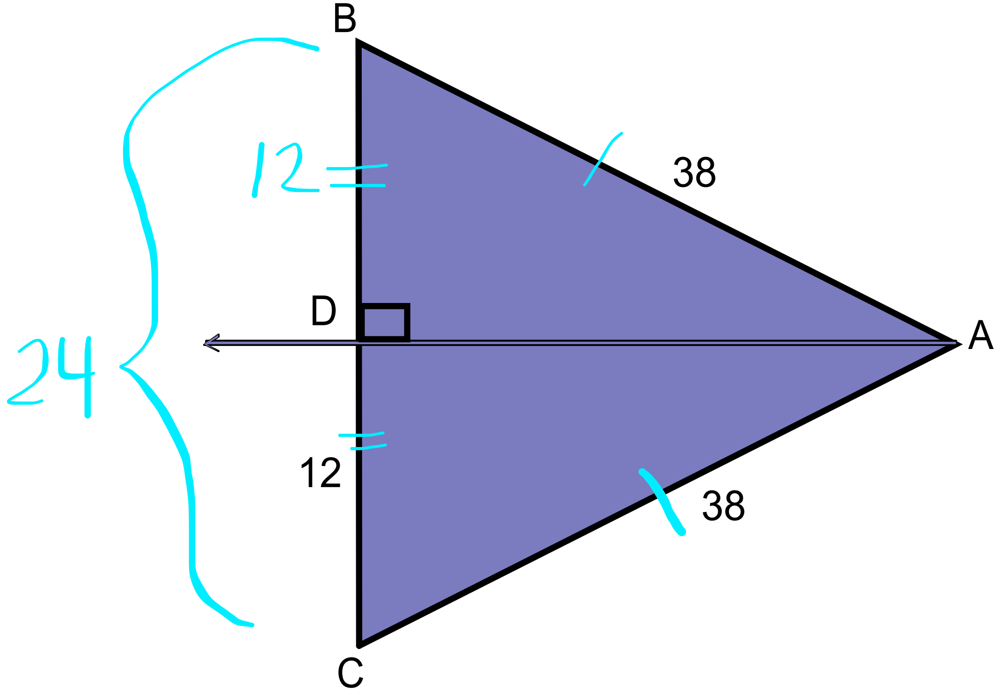
S is the incenter.

$\overline{QS}$ is an $\angle$ Bisector!

- What is segment AD and why? Find the measure of BC.

$\overline{AD}$ is all 4 segments!

Isos. △



Hint: Move triangle to see what segment AD is and then drag BC to triangle to find its measure.

Applying and Solving

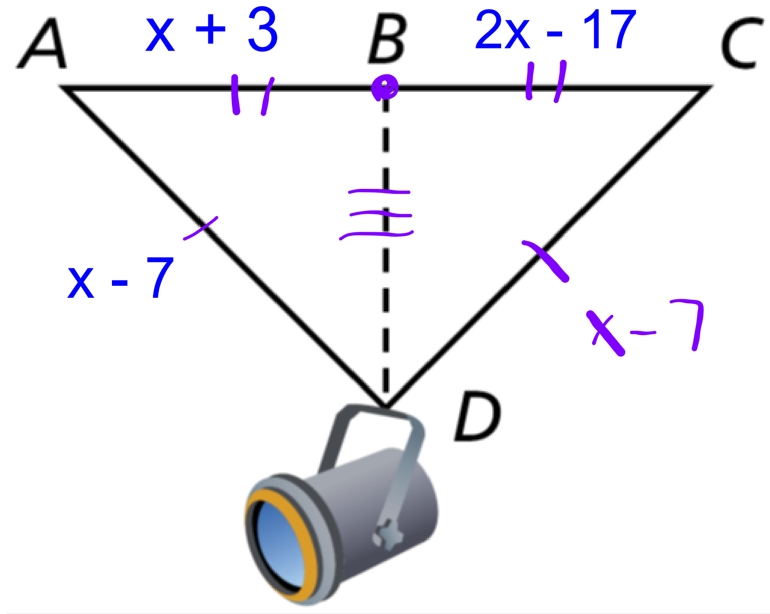
John wants to hang a spotlight along the back of a display case. Wires *AD* and *CD* are the same length, and *A* and *C* are equidistant from *B*. How long are AD and CD? Justify your answer.

$\overline{BD}$ is a median.

$x+3=2x-17$

$20=x$

$20-7=\boxed{13u}$



Where is the center of the triangle?

3 perpendicular bisectors.gsp

3 altitudes.gsp

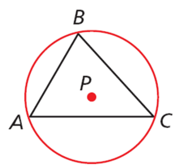
3 medians.gsp

3 angle bisectors.gsp

	Perpendicular Bisector	Altitude	Median	Angle Bisector
Acute Triangle	In	In	In	In
Right Triangle	On	On	In	In
Obtuse Triangle	Outside	Outside	In	In

Point of Concurrency:

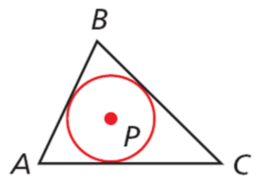
Circumcenter - = from vertices



Orthocenter

Centroid - center of gravity

Incenter - = from sides



An Odd Peanut Butter Cup May Contain Apple Butter Instead

Altitude Orthocenter, Perpendicular Bisector Circumcenter, Median Centroid, Angle Bisector Incenter

aopbcmcabi.gsp

Altitude: A segment that has an endpoint at a vertex of a triangle and the other is on the side opposite the vertex and $\perp$ to this line. (The altitude may lie on the outside or inside of the triangle.)

POC
Orthocenter: The intersection of the altitudes of a triangle.

Perpendicular Bisector: A line or line segment that passes through the mdpt of a side of a triangle *and* is $\perp$ to that side.

POC
Circumcenter: Point of intersection of the Perpendicular Bisector of a triangle. The circumcenter is equidistant from the vertices of the triangle. PPW

Median: A segment that connects a vertex of a triangle to the mdpt of the side opposite the vertex.

POC
Centroid: Point of intersection of the medians of a triangle. The centroid is 2/3 of the distance from each vertex to the midpoint of the opposite side. Balance/gravity

Angle Bisector: A segment that splits in ½ an angle of the triangle and has one endpoint at a vertex of the triangle and the other on another point on the triangle.

POC
Incenter: The intersection of the ∠ Bisectors of a triangle. The incenter is equidistant from the three sides of the triangle.

Special Triangles:

Isosceles Triangle: The median, angle bisector, altitude, and perpendicular bisector from the same vertex is the same segment. The centroid, incenter, orthocenter, and circumcenter will be collinear.

Equilateral Triangle: The medians, angle bisectors, altitudes, and perpendicular bisectors from each vertex form three segments on the interior of the triangle. The centroid, incenter, orthocenter, and circumcenter are all the same point.

Right Triangle: Two of the altitudes are the legs of the triangle. The orthocenter lies on the vertex of the right angle.

Euler Segment: The segment formed by connecting the Centroid, Orthocenter and Circumcenter. (They are always collinear)

6.1: pg. 306: 3, 7, 11, 15, 19, 23, 25, 39 - 44

6.2: pg. 315: 3, 5, 7, 11, 25, 29, 31, 52 - 59

6.3: pg. 324: 3, 7, 11, 15, 19, 27, 31, 33, 35, 55 - 58

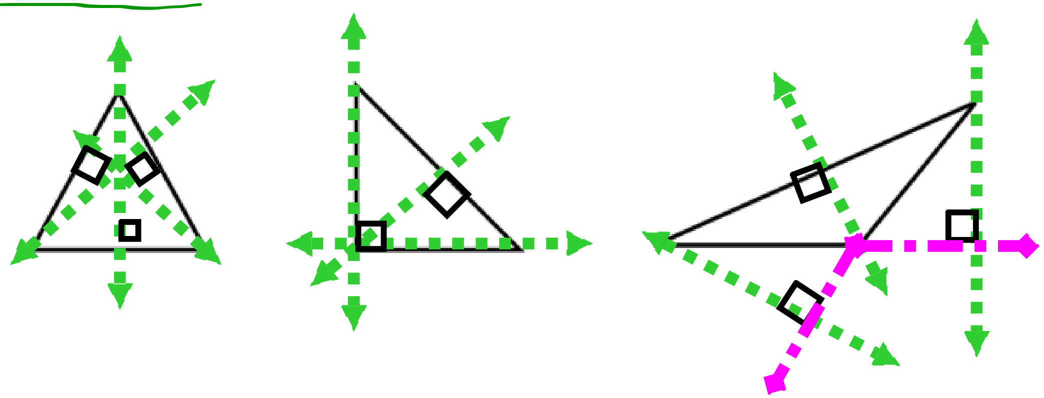
These practice problems are due Friday in your folder.

ALTITUDES

1. Define altitude of a triangle. Segment from the vertex to opposite side perpendicularly.
2. How many altitudes can a triangle have? 3
3. What is the name of the point of intersections of the altitudes? Orthocenter
4. Do they always intersect inside the triangle? NO Why or why not?
   Inside, On, and Outside depending on type of triangle
5. Draw an example of three triangles (acute, right, and obtuse) with all three altitudes.
   Below

Altitude: A segment that has an endpoint at a vertex of a triangle and the other is on the side opposite the vertex and ⊥ perpendicular to this line. (The altitude may lie on the outside or inside of the triangle.)

POC **Orthocenter**: The intersection of the altitude of a triangle.

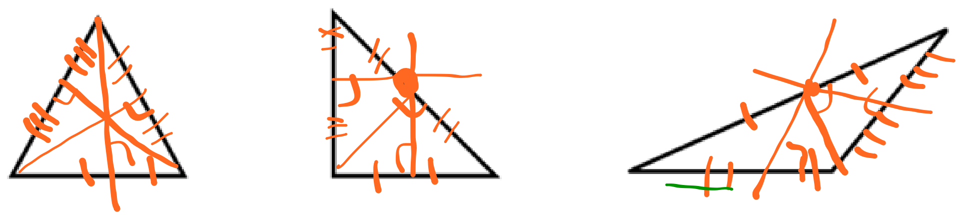


PERPENDICULAR BISECTORS

1. Describe how to draw a perpendicular bisector of a triangle. Segment to opp. side perpendicular through the midpoint.
2. How many perpendicular bisectors can a triangle have? 3
3. What is the name of the point of concurrency of the perpendicular bisectors? Circumcenter
4. Do they always meet inside the triangle? NO Why or why not? Can intersect inside, on, or outside
5. Make a conjecture about the distance from the angles of the triangle to the point of intersection. Equal distance
   Explain why this conjecture happens. splits the sides of the triangle too
6. What is the significance of the point of intersection of the perpendicular bisectors? Equidistant from vertices to circumcenter
7. Draw an example of three triangles (acute, right, and obtuse) with all three angle bisectors. below

Perpendicular Bisector: A line or line segment that passes through the midpoint of a side of a triangle *and* is ⊥ perpendicular to that side.

POC **Circumcenter**: Point of intersection of the perpendicular bisectors of a triangle. The circumcenter is equidistant from the vertices of the triangle.

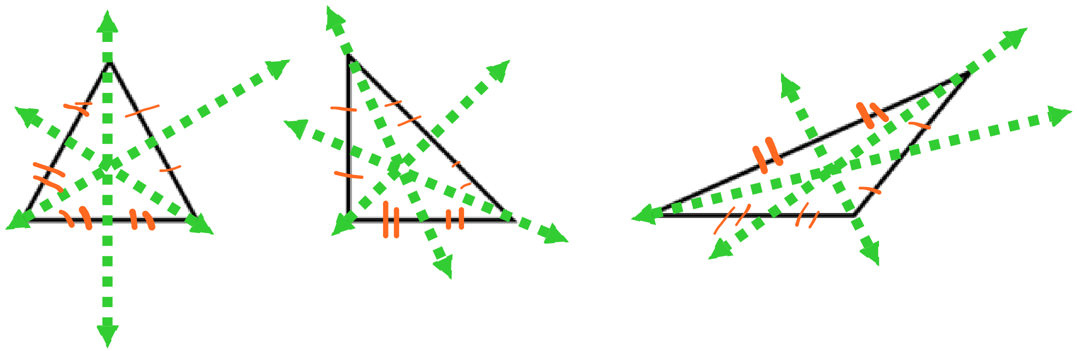


MEDIANS

1. What is a median of a triangle? Segment from vertex to midpoint of opposite side.
2. How many medians can a triangle have? 3
3. What is the name of the point of intersection of the medians? Centroid
4. Do they always meet inside the triangle? Yes Why or Why not? balancing point of a triangle must be inside.
5. Make a conjecture about the length of the segment from a vertex to the point of concurrency in relation to the entire length of the median. distance is two-thirds the way from the vertex to the midpoint.
6. What is the significance of the point of concurrency of the medians? balancing point
7. Draw an example of three triangles (acute, right and obtuse) with three medians. below

Median: A segment that connects a vertex of a triangle to the midpoint of the side opposite the vertex.

POC **Centroid**: Point of intersection of the medians of a triangle. The centroid is 2/3 of the distance from each vertex to the midpoint of the opposite side.



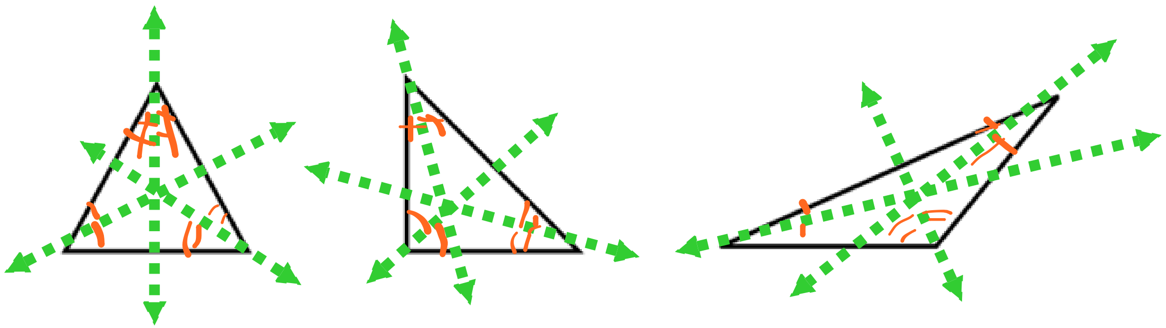
ANGLE BISECTORS

1. Describe how to draw an angle bisector of a triangle. segment from vertex to opposite side while splitting angle in half.
2. How many angle bisectors can a triangle have? 3
3. What is the name of the point of concurrency of the angle bisectors? Incenter
4. Do they always meet inside the triangle? Yes Why or why not? equidistant from POC to sides
5. Make a conjecture about the distance from the sides of the triangle to the point of intersection. equidistant from POC to sides
   Explain why this conjecture happens.
6. What is the significance of the point of intersection of the angle bisectors? equidistant from POC to sides
7. Draw an example of three triangles (acute, right, and obtuse) with all three angle bisectors. below

Angle Bisector: A segment that bisects an angle of the triangle and has one endpoint at a vertex of the triangle and the other on another point on the triangle.

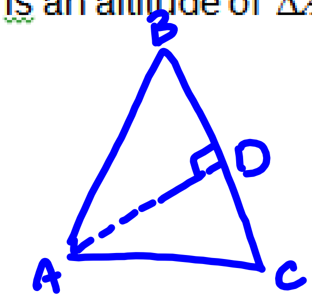
POC

Incenter: The intersection of the Angle Bisector of a triangle. The incenter is equidistant from the three sides of the triangle.

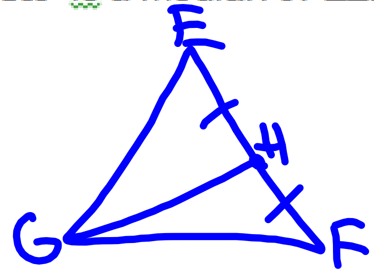


Draw and label a figure to illustrate each situation. Be sure to include appropriate markings.

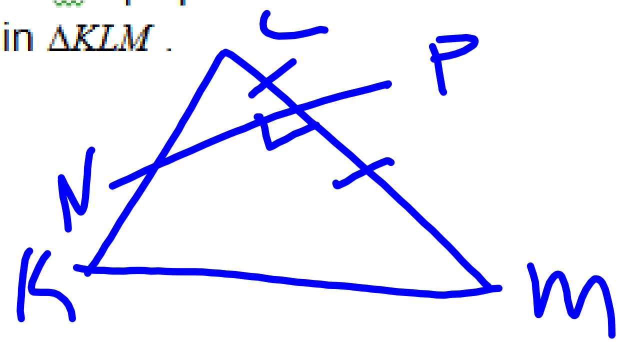
1. $\overline{AD}$ is an altitude of ΔABC.



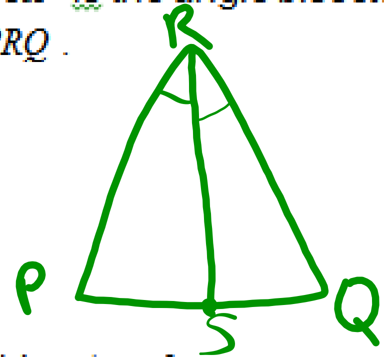
2. $\overline{GH}$ is a median of ΔEFG.



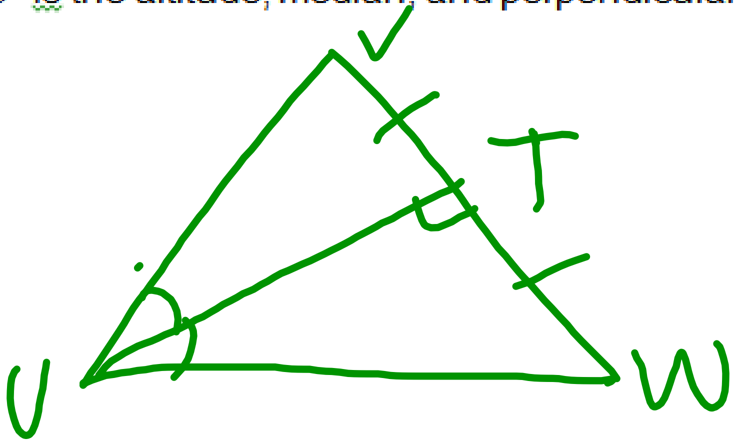
3. $\overline{NP}$ is a perpendicular bisector of $\overline{ML}$ in ΔKLM.



4. $\overline{RS}$ is the angle bisector of ΔPRQ.



5. $\overline{TU}$ is the altitude, median, and perpendicular bisector of ΔUVW.



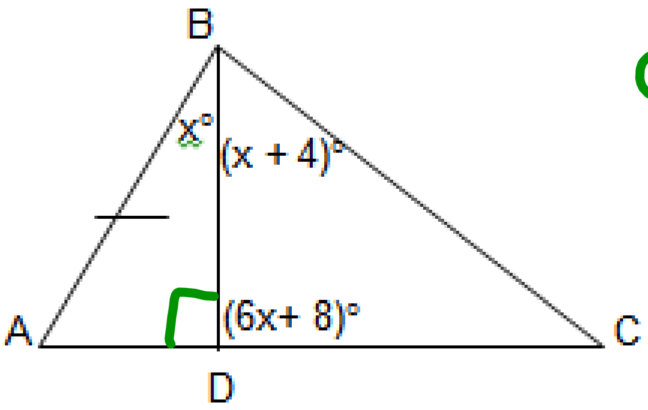
Answer the following with Always, Sometimes or Never.

6. The three altitudes of a triangle intersect at a vertex of the triangle. S

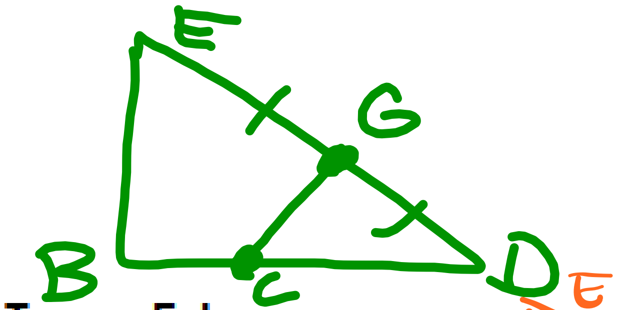
7. The three medians of a triangle intersect at a point outside the triangle. N

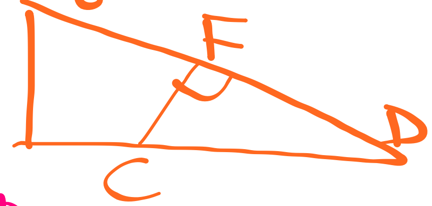
8. The three angle bisectors of a triangle intersect at a point inside the triangle. A

9. Find the value of x if $\overline{BD}$ is an altitude of ΔABC. $\frac{41}{3}$



$$90 = 6x + 8$$
$$-8 \qquad -8$$
$$\frac{82}{6} = \frac{\cancel{6}x}{\cancel{6}}$$

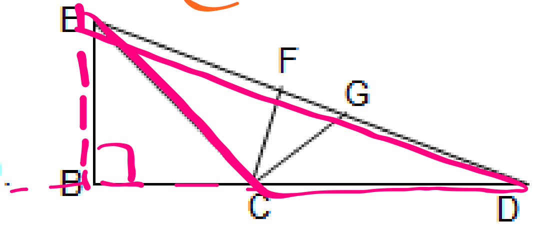




Use the picture to the right to determine True or False:

False 10. If G is the midpoint of $\overline{ED}$, then $\overline{CG}$ is a median of $\triangle EBD$.

True 11. If $\overline{CF} \perp \overline{ED}$, then $\overline{CF}$ is an altitude of both $\triangle ECD$ and $\triangle ECG$.



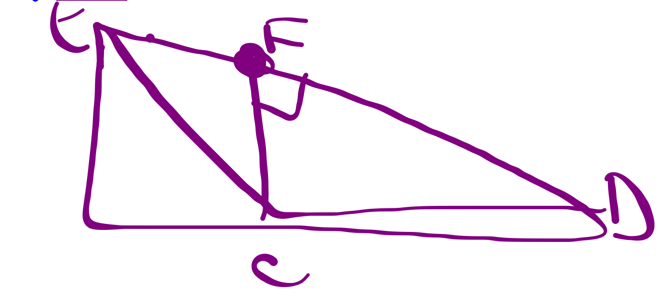
True 12 If $\overline{EB} \perp \overline{BD}$, the $\overline{EB}$ is an altitude of $\triangle ECD$.

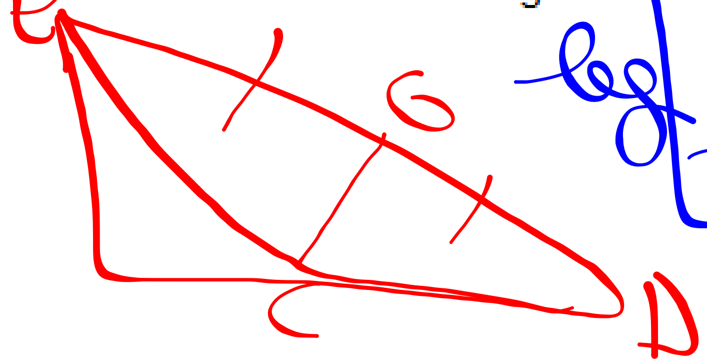
False 13. If $\overline{CF} \perp \overline{ED}$, then $\overline{CF}$ is a perpendicular bisector of $\triangle ECD$.

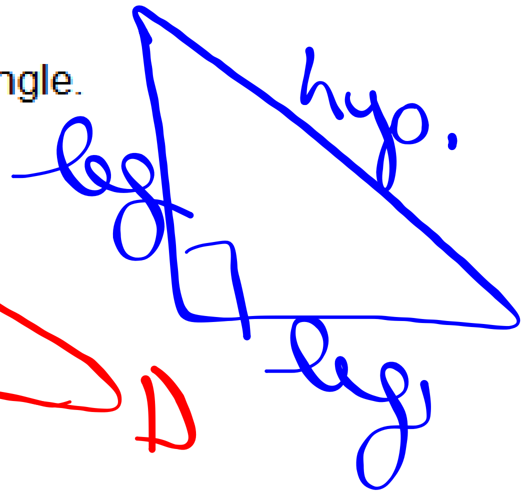
$\overline{EF} \cong \overline{FD}$

True 14. If $\overline{CG}$ is a median of $\triangle ECD$, then G is the midpoint of $\overline{ED}$.

True 15. Each leg of a right triangle is also an altitude of the triangle.







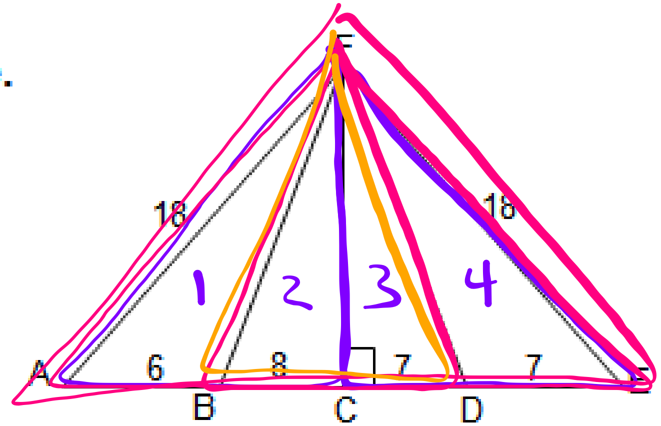
Complete each statement in as many ways as possible.

16. $\overline{FD}$ is median of $\triangle$FCE. (1 answer)

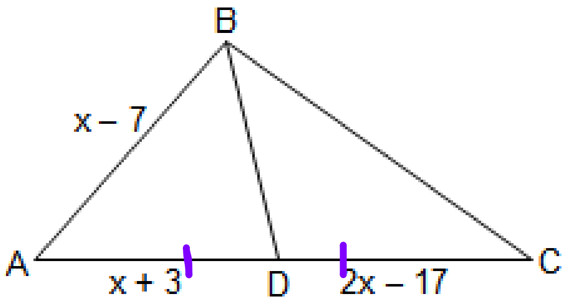
17. $\overline{FC}$ is $\perp$ bisector, altitude, $\measuredangle$ bisector, Median of $\triangle$AFE. (4 answers)

18. $\overline{FC}$ is altitude of $\triangle$BFE. (1 answer)

19. $\overline{FC}$ is an altitude of 10 triangles.



20. Find $\overline{AB}$ if $\overline{BD}$ is a median of $\triangle ABC$.

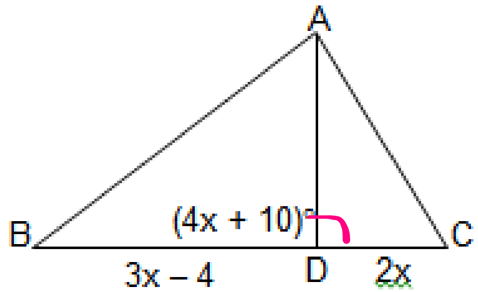


$x+3 = 2x-17$

$20 = x$

$20-7 = \boxed{13}$

21. Find $\overline{BC}$ if $\overline{AD}$ is an altitude of $\triangle ABC$.



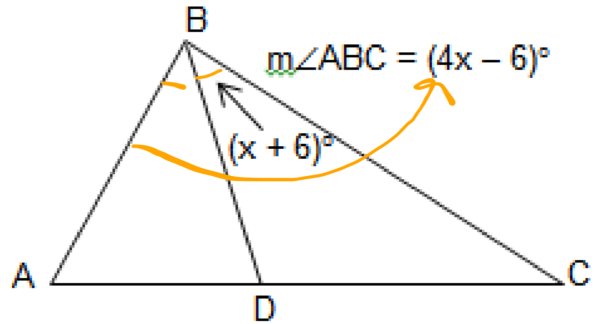
$4x+10=90$

$x=20$

$5(20)-4$

$\boxed{96}$

22. Find $m\angle ABC$ if $\overline{BD}$ is an angle bisector of $\triangle ABC$.



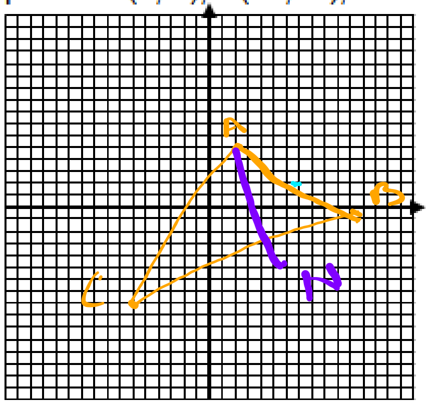
$2(x+6) = 4x-6$

$x+6 = 2x-3$

$9 = x$

$4(9)-6 = \boxed{30°}$

23. Plot the points. A(2, 5), B(12, -1), and C(-6, -8) are the vertices of Δ ABC.



24. What are the coordinates of K if $\overline{CK}$ is a median of △ABC?

midpt of AB: $\left(\frac{2+12}{2}, \frac{5-1}{2}\right) = \boxed{(7, 2)}$

25. What is the slope of the perpendicular bisector of $\overline{AB}$?

m of $\overline{AB}$ $\frac{\Delta y}{\Delta x} = \frac{-1-5}{12-2} = \frac{-6}{14} = -\frac{3}{7} \rightarrow \boxed{\perp m = -\frac{7}{3}}$

26. What is the slope of $\overline{CL}$ if $\overline{CL}$ is the altitude from point C?

m of $\overline{AB} = -\frac{3}{7}$ $\boxed{CL = -\frac{7}{3}}$

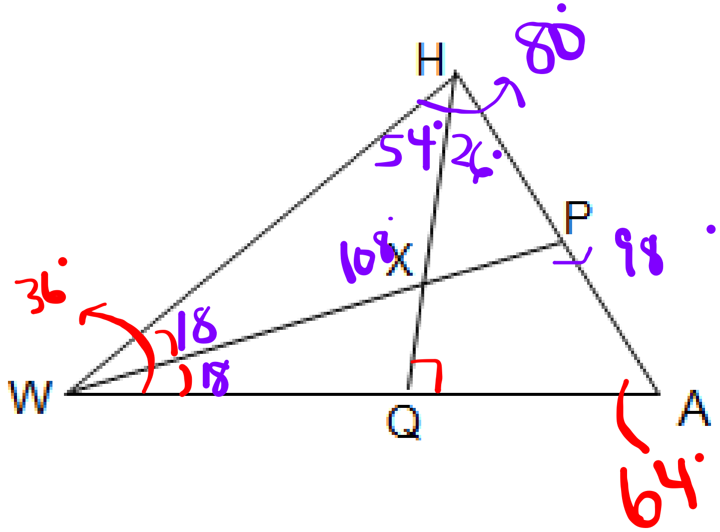
27. Point N on $\overline{BC}$ has coordinates $(6, \frac{-10}{3})$. Is $\overline{NA}$ an altitude of △ABC? Explain your answer.

m $\overline{AN}$: $\frac{5+\frac{10}{3}}{2-6} = \frac{\frac{25}{3}}{-4} = \frac{25}{3} \cdot \frac{1}{-4} = \frac{25}{-12}$

m $\overline{BC} = \frac{-1+8}{12+6} = \frac{7}{18}$

$\boxed{\text{No, not } \perp \text{ m's}}$

In $\triangle AHW$, $m\angle A = 64°$ and $m\angle AWH = 36°$. If $\overline{WP}$ is an angle bisector and $\overline{HQ}$ is an altitude, find each measure.



28. $m\angle AQH =$ 90°

29. $m\angle APW =$ 98°

30. $m\angle AHQ =$ 26°

31. $m\angle HXW =$ 108

32. If $\overline{WP}$ is a median, $AP = 3y + 11$ and $PH = 7y - 5$, find AH.

$3y + 11 = 7y - 5$

$16 = 4y$

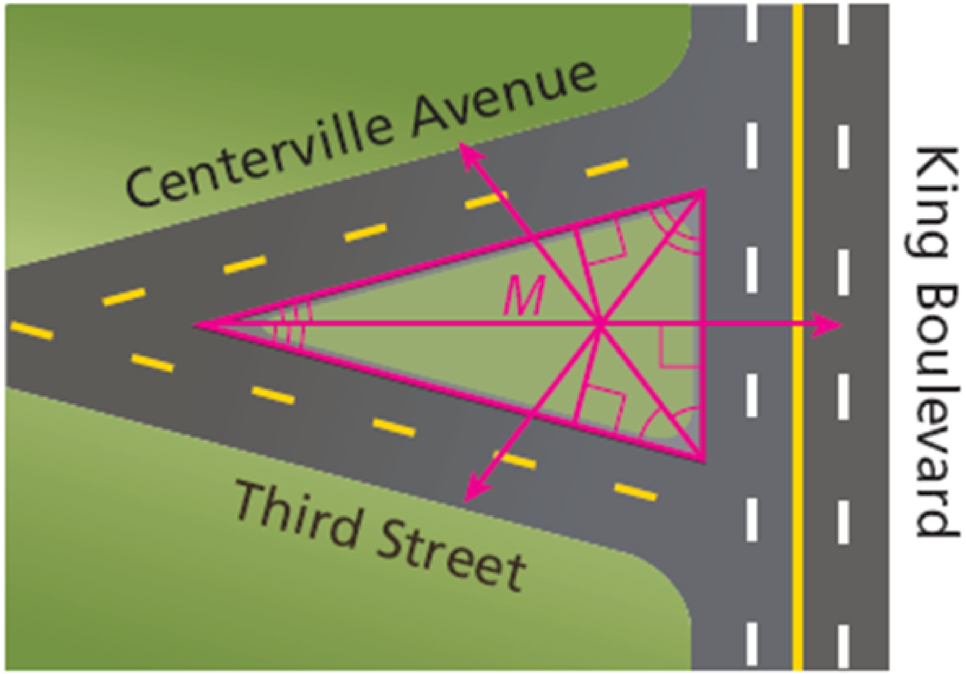
$y = 4$

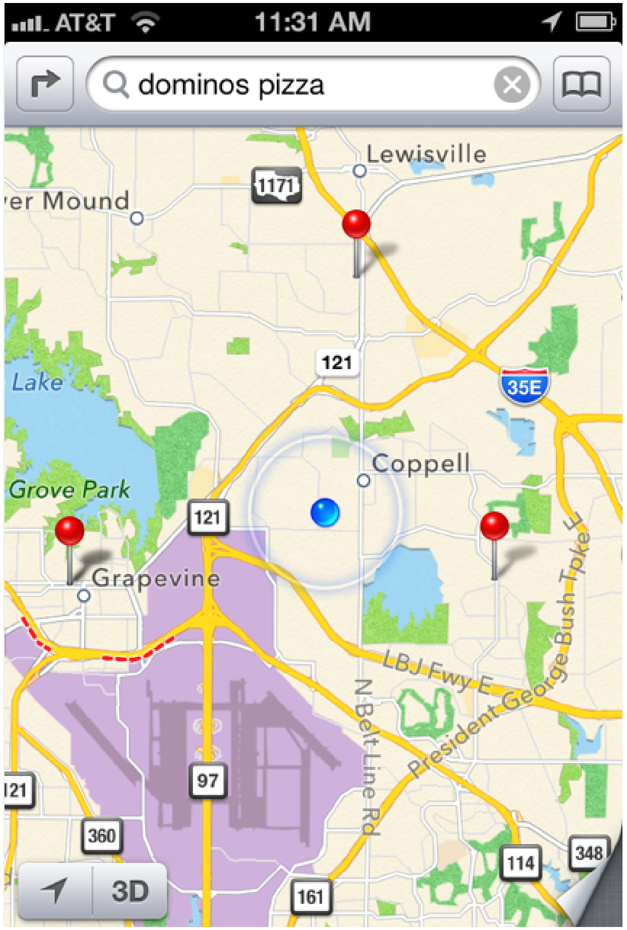
$2[3(4) + 11]$

$2[12 + 11] = 2(23) = \boxed{46}$

Example Scenarios for the Partner Project

In 2012 Coppell planned to build a volleyball state championship monument in the park between three streets. Where should the city place the monument so that it is the same distance from all three streets.

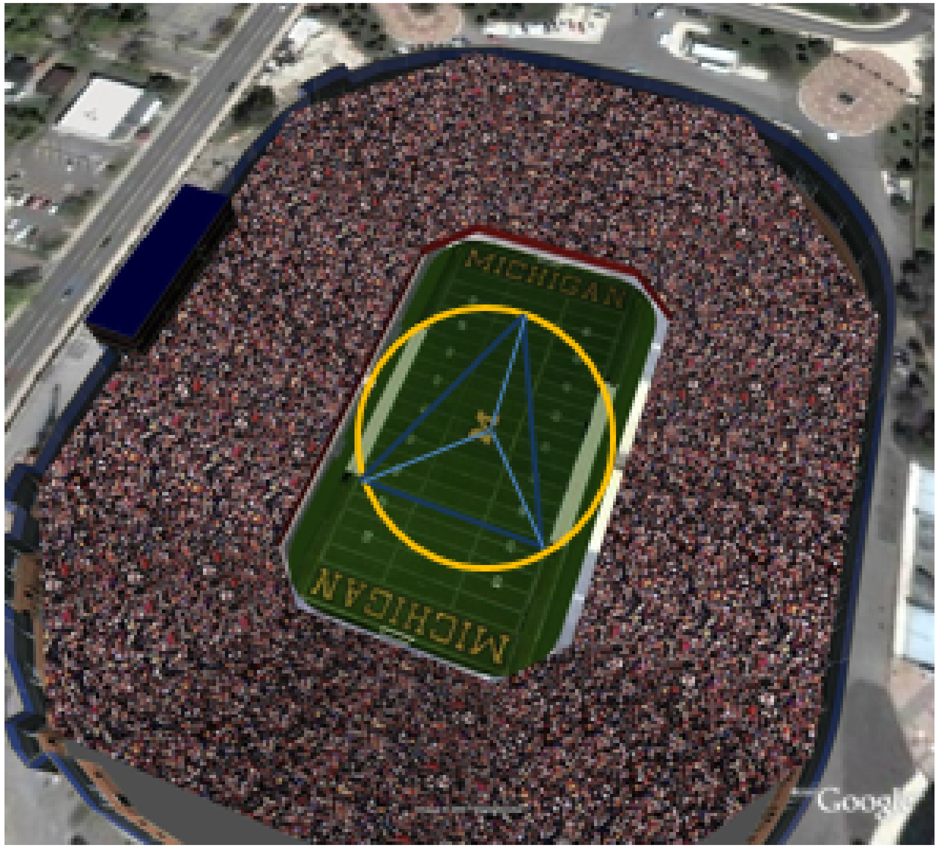




True story!
What does my house appear to represent IF all dominos are equidistant from me?

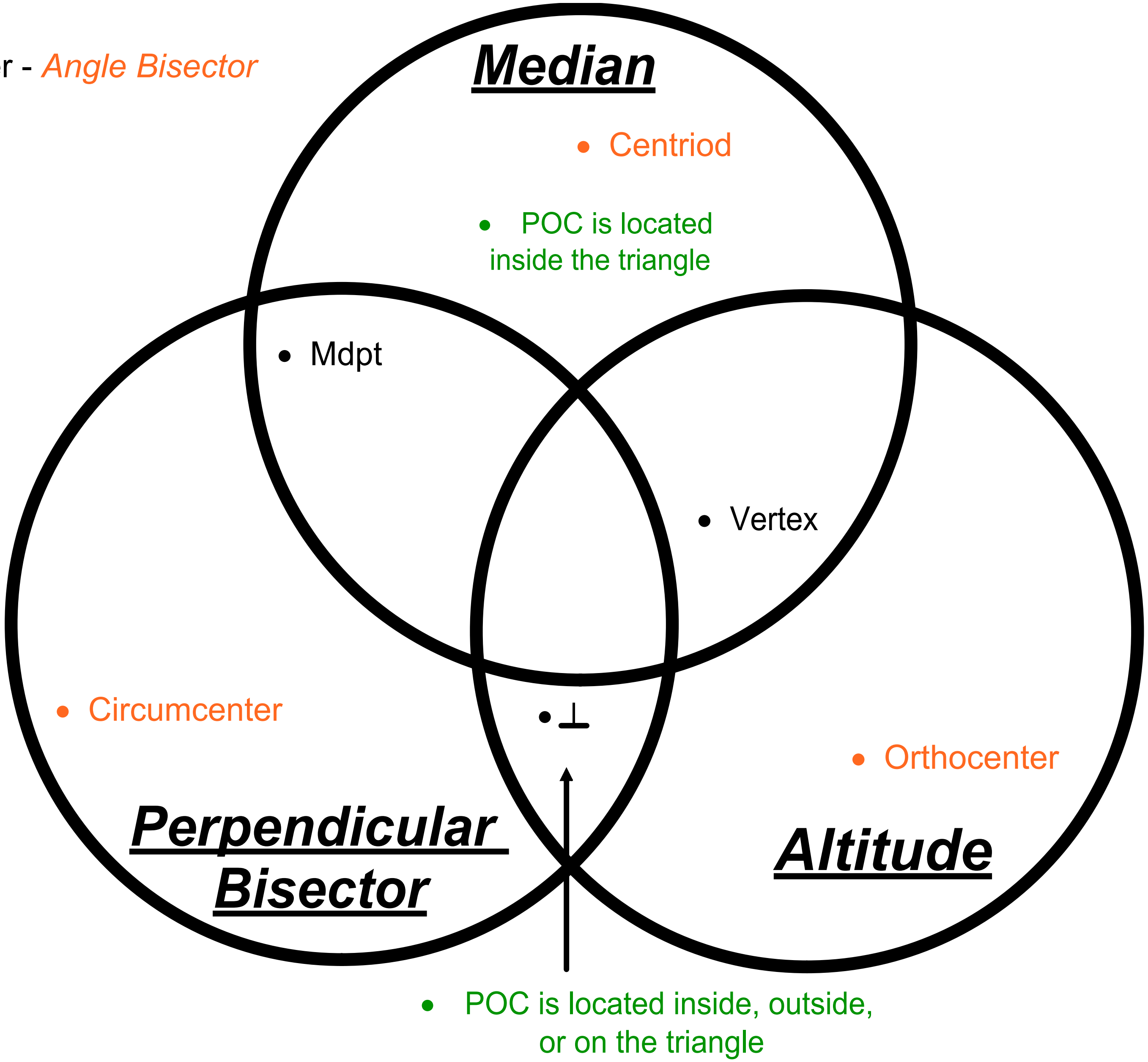
Will the solar car flip over??

http://www.youtube.com "Coppell's Solar Car"

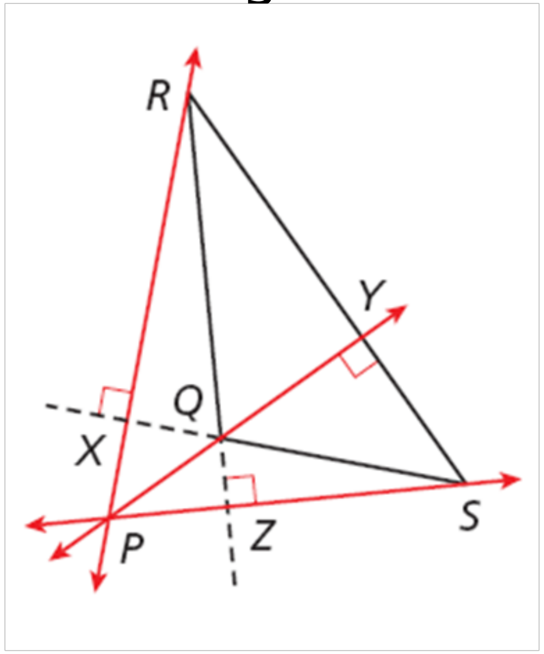


In order to find where to paint the U of M logo on the football field, they found the circumcenter of a triangle whose vertices were on a circle. The equal distance from the vertices show the circumcenter.

- Incenter - *Angle Bisector*

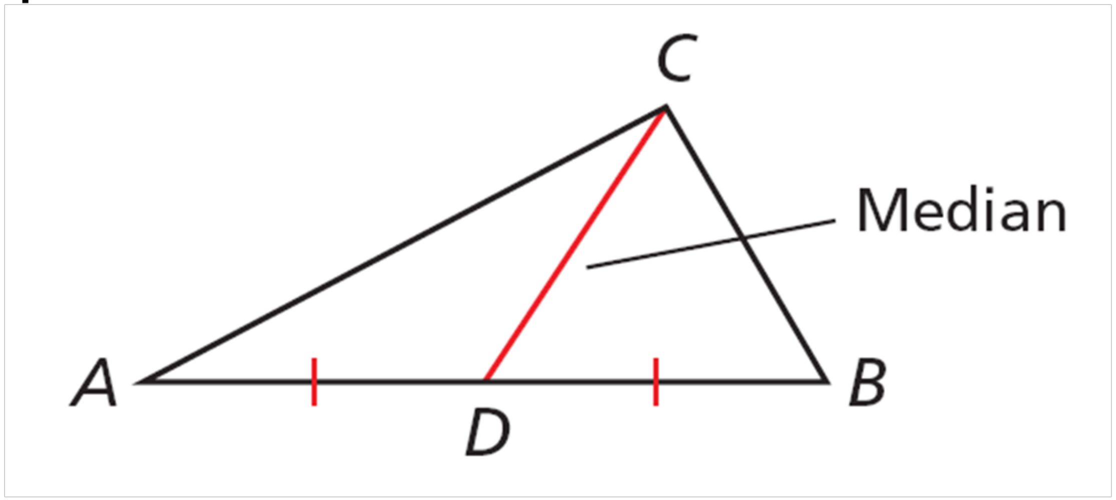


An **<u>altitude of a triangle</u>** is a perpendicular segment from a vertex to the line containing the opposite side.



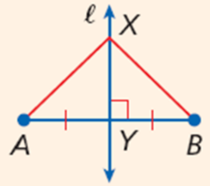
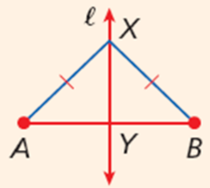
Every triangle has <u>three</u> altitudes. An altitude can be *inside*, *outside*, or *on* the triangle.

A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.



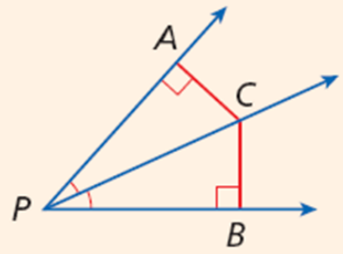
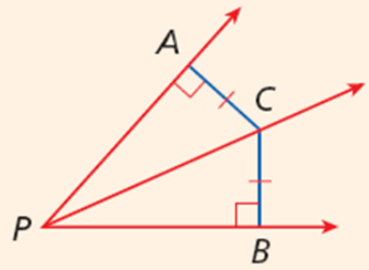
Every triangle has three medians, and the medians are concurrent.

Theorems — Distance and Perpendicular Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
5-1-1 Perpendicular Bisector Theorem If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	 $\overline{XY} \perp \overline{AB}$ $\overline{YA} \cong \overline{YB}$	$XA = XB$
5-1-2 Converse of the Perpendicular Bisector Theorem If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	 $XA = XB$	$\overline{XY} \perp \overline{AB}$ $\overline{YA} \cong \overline{YB}$

Remember that the distance between a point and a line is the length of the perpendicular segment from the point to the line.

Theorems — Distance and Angle Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
5-1-3 **Angle Bisector Theorem** If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 $\angle APC \cong \angle BPC$	$AC = BC$
5-1-4 **Converse of the Angle Bisector Theorem** If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 $AC = BC$	$\angle APC \cong \angle BPC$

Attachments

aopbcmcabi.gsp

Altitudes copy test.gsp

perpendicular bisectors.gsp

Angle bisectors.gsp

Altitudes copy.gsp

medians.gsp

3 medians.gsp

3 altitudes.gsp

3 perpendicular bisectors.gsp

3 angle bisectors.gsp

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