6.1 Properties of Exponents DAY ONE

Essential Question: How can you write general rules involving properties of exponents?

What you will learn:

- use zero and negative exponents.
- use the properties of exponents.
- solve real-life problems involving exponents.

Core Vocabulary:

- power
- exponent
- base
- scientific notation

EXPLORATION: Writing Rules for Properties of Exponents

1. What happens when you multiply two powers with the same base? Write the product of two powers as a single power. Then write a *general rule* for finding the product of two powers with the same base.

a) $(2^2)(2^3)$ = 2^5

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

b) $(4^1)(4^5)$ = 4^6

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

c) $(5^3)(5^5)$ = 5^8

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

d) $(x^2)(x^6)$ = x^8

$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

general rule: $b^m \cdot b^n = b^{m+n}$

2. What happens when you divide two powers with the same base? Write the quotient of the two powers as a single power. Then write a *general rule* for finding the quotient of two powers with the same base.

a) $\frac{4^3}{4^2} = \frac{4\cdot4\cdot4}{4\cdot4} = 4$

b) $\frac{2^5}{2^2} = \frac{2\cdot2\cdot2\cdot2\cdot2}{2\cdot2} = 2^3 = 8$

c) $\frac{x^6}{x^3} = \frac{x\cdot x\cdot x\cdot x\cdot x\cdot x}{x\cdot x\cdot x} = x^3$

d) $\frac{3^4}{3^4} = \frac{3\cdot3\cdot3\cdot3}{3\cdot3\cdot3\cdot3} = 3^0 = 1$

general rule: $\frac{b^m}{b^n} = b^{m-n}$

3. What happens when you find a power of a power? Write the expression as a single power. Then write a *general rule* for finding a power of a power.

a) $(2^2)^4 = 2^{2\cdot4} = 2^8 = 256$

$2^2\cdot2^2\cdot2^2\cdot2^2$

22222222

b) $(7^3)^2 = 7^{3\cdot2} = 7^6 = 117649$

$7^3\cdot7^3$

$777\cdot777$

c) $(y^3)^3 = y^{3\cdot3} = y^9$

$y^3\cdot y^3\cdot y^3$

$yyy\cdot yyy\cdot yyy$

d) $(x^4)^2 = x^{4\cdot2} = x^8$

general rule: $(b^n)^n = b^{m\cdot n}$

Using the Properties of Exponents

REMEMBER:

the expression x^3 is called a power. The base, x, is used as a factor 3 times because the exponent is 3.

integer is a whole number; a number that is not a fraction.

	Property	Words	Numbers	Formula
Product of Powers	Let a be a real number, and let m and n be integers.	To multiply powers with the same base, add their exponents.	$4^6 * 4^3 = 4^{6+3} = 4^9$	$a^m * a^n = a^{m+n}$
Quotient of Powers	Let a be a nonzero real number, and let m and n be integers.	To divide powers with the same base, subtract their exponents.	$\frac{4^6}{4^3} = 4^{6-3} = 4^3$	$\frac{a^m}{a^n} = a^{m-n}; a \neq 0$
Power of a Power	Let a be a real number, and let m and n be integers.	To find a power of a power, multiply the exponents.	$(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$	$(a^m)^n = a^{m \cdot n}$
Zero Exponent	Let this apply any time there is an exponent of zero.	For any nonzero number a, $a^0 = 1$. The power 0^0 is undefined.	$4^0 = 1$	$a^0 = 1$, where $a \neq 0$.
Negative Exponent	Let this apply any time there is a negative number in the exponent.	For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$	$a^{-n} = \frac{1}{a^n}, a \neq 0$

Examples: Evaluate each expression.

a) $6.7^0 = 1$

b) $(-2)^{-4}$ $\frac{1}{(-2)^4} = \frac{1}{(-2)(-2)(-2)(-2)} = \frac{1}{16}$

c) $(2^2)^{-3} = 2^{2 \cdot -3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{222222} = \frac{1}{64}$

d) $x^1(x^1) = x^{1+1} = x^2$

More practice:

Simplify the expression. Then write your answer using only positive exponents.

$\frac{2^0 x^2}{y^{-2}} = \frac{(1)x^2y^2}{} = x^2y^2$

$\frac{4x^0}{y^{-3}} = \frac{4(1)(y^3)}{} = 4y^3$

Your Turn:

Simplify the expression. Then write your answer only using positive exponents.

1) $(-9)^0 = 1$

2) $3^{-3} = \frac{1}{3^3} = \frac{1}{3 \cdot 3 \cdot 3} = \boxed{\frac{1}{27}}$

3) $\frac{-5^0}{2^{-2}} = -(1)2^2$
$= -2 \cdot 2$
$= \boxed{-4}$

4) $\frac{3^{-2}x^{-5}}{y^0} = \frac{1}{3^2(1)x^5} = \frac{1}{3 \cdot 3x^5}$
$= \boxed{\frac{1}{9x^5}}$

Simplify each expression. Write your answer using only positive exponents.

a) $3^2 \bullet 3^6$
3^{2+6}
3^8
$\boxed{6561}$

b) $\frac{(-4)^2}{(-4)^7}$
$= (-4)^{2-7}$
$= (-4)^{-5}$
$= \frac{1}{(-4)^5}$
$= \frac{1}{(-4)(-4)(-4)(-4)(-4)}$
$= \boxed{\frac{1}{-1024}}$

c) $(z^4)^{-3}$
$= z^{4 \cdot -3}$
$= z^{-12}$
$= \boxed{\frac{1}{z^{12}}}$

MORE PRACTICE:

a) $(10y)^{-3} = 10^{-3}y^{-3}$

$= \frac{1}{10^3y^3} = \frac{1}{10\cdot10\cdot10\,y^3} = \boxed{\frac{1}{1000y^3}}$

b) $\left(\frac{a}{-10}\right)^3 = \frac{a^3}{(-10)^3}$

$= \frac{a^3}{(-10)(-10)(-10)} = \boxed{\frac{a^3}{-1000}}$

c) $\left(\frac{3d}{2}\right)^4 = \frac{3^4d^4}{2^4}$

$= \frac{3333d^4}{2\cdot2\cdot2\cdot2} = \boxed{\frac{81d^4}{16}}$

d) $\left(\frac{2x}{3}\right)^{-5} = \left(\frac{3}{2x}\right)^5 = \frac{3^5}{2^5x^5}$

$= \frac{33333}{22222x^5} = \boxed{\frac{243}{32x^5}}$

YOUR TURN:

1) $(-1.5y)^2 = (-1)^2(1.5)^2(y)^2$

$= (1)(2.25)y^2$

$= \boxed{2.25y^2}$

2) $\left(-\frac{4}{n}\right)^5 = \frac{(-1)^5(4)^5}{n^5} = \frac{-1(4^5)}{n^5}$

$= \boxed{\frac{-1024}{n^5}}$

3) $\left(\frac{1}{2k^2}\right)^5 = \frac{1^5}{2^5k^{2\cdot5}}$

$= \boxed{\frac{1}{32k^{10}}}$

4) $\left(\frac{6c}{7}\right)^{-2} = \left(\frac{7}{6c}\right)^2$

$= \frac{7^2}{6^2c^2} = \boxed{\frac{49}{36c^2}}$

Real-Life Examples

1) Which of the expressions shown represent the volume of the cylinder, where r is the radius and h is the height?

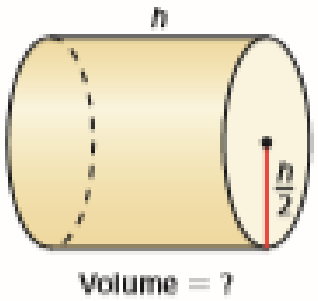
$V = B \cdot h$

$B = \pi r^2$

$= \pi r^2 \cdot h$

$= \pi \left(\frac{h}{2}\right)^2 \cdot h$

$= \frac{1}{4}\pi h^3$



$2\pi r^3$ ~~$\pi h^3 2^{-2}$~~ πh^{-1}

$\frac{\pi h^2}{4}$ $\frac{\pi h^3}{4}$ $\frac{\pi h^{-1}}{2}$

2) It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the center of the Milky Way? Write your answer in scientific notation.

$(2.3 \times 10^8) \cdot (2.5 \times 10^2)$

$(2.3)(2.5)(10^8)(10^2)$

5.75×10^{10}

57,500,000,000

YOUR TURN:

Which of the expressions shown represent the volume of the cone, where r is the radius and h is the height?

$\frac{\pi h^3}{12}$ $\frac{\pi r^3}{3}$ $\frac{\pi h^2}{6}$

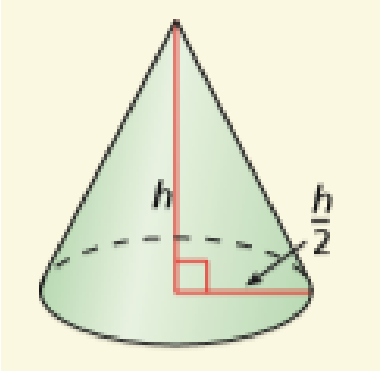
$\frac{\pi h^2}{12}$ $\frac{2\pi r^3}{3}$ $8\pi r^3$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h^1$

$= \frac{1}{3} \cdot \frac{1}{4}\pi \cdot h^3$

$= \boxed{\frac{1}{12}\pi h^3}$



Assignment for Day One of

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Extra Practice WS