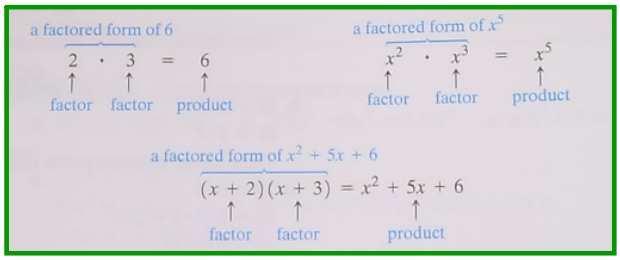
6.1 The Greatest Common Factor & Factoring by Grouping DAY ONE

REVIEW

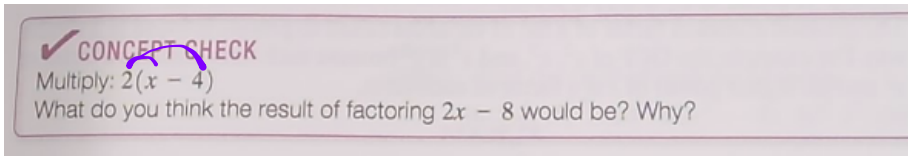


Product (2)(3) = 6

2 & 3 are factors of 6

(2)(3) is in factored form of 6

The process of writing a polynomial as a product is called factoring the polynomial.



$2x - 8$

$(2)(x-4)$

OBJECTIVE 1: Finding the Greatest Factor of a List of Integers

The first step in factoring a polynomial is to see whether the terms of the polynomial have a common factor (GCF).

If there is one then write the polynomial as a product by factoring out the common factor.

This term factored out is called the greatest common factor or GCF.

Finding the GCF of a List of Integers

Step 1. Write each number as a product of prime numbers.

Step 2. Identify the common prime factors.

Step 3. The product of all common prime factors found in Step 2 is the greatest common factor. If there are no common prime factors, the greatest common factor is 1.

Example 1: Find the GCF of each list of numbers.

a) 28 and 40

$4 \cdot 7$ $\quad$ $4 \cdot 10$

$2 \cdot 2$ $\quad$ $2 \cdot 2 \cdot 2 \cdot 5$

$2 \cdot 2 \cdot 7$ $\quad$ $2 \cdot 2 \cdot 2 \cdot 5$

$2^2 \cdot 7$ $\quad$ $2^3 \cdot 5$

$2^2 = 4$

b) 55 and 21

$5 \cdot 11$ $\quad$ $3 \cdot 7$

$5 \cdot 11$ $\quad$ $3 \cdot 7$

1

c) 15, 18, and 66

$3 \cdot 5$ $\quad$ $3^2 \cdot 2$ $\quad$ $2 \cdot 3 \cdot 11$

3

Practice 1:

a) 36 and 42

$4 \cdot 9$ $\quad$ $6 \cdot 7$

$2 \cdot 2$ $\quad$ $3 \cdot 3$ $\quad$ $2 \cdot 3$

$2^2 \cdot 3^2$ $\quad$ $2 \cdot 3 \cdot 7$

$2 \cdot 3 = 6$

b) 35 and 44

$5 \cdot 7$ $\quad$ $4 \cdot 11$

$5 \cdot 7$ $\quad$ $2^2 \cdot 11$

1

c) 12, 16, and 40

$4 \cdot 3$ $\quad$ $4 \cdot 4$ $\quad$ $4 \cdot 10$

$2^2 \cdot 3$ $\quad$ 2^4 $\quad$ $2^3 \cdot 5$

$2^2 = 4$

OBJECTIVE 2: Finding the Greatest Common Factor of a List of Terms

The greatest common factor of a list of variables raised to powers is found in a similar way.

$$x^2 = x \cdot x$$

$$x^3 = x \cdot x \cdot x$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Example 2: Find the GCF of each list of terms.

a) x^3, x^7, and x^5

x^3

b) y, y^4, and y^7

y

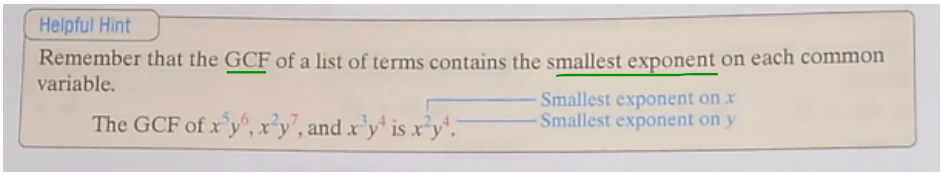
Practice 2:

a) y^6, y^4, and y^7

y^4

b) x, x^4, and x^2

x



Example 3: Find the GCF of each list of terms.

a) $6x^2$, $10x^3$, & $-8x$

$2\cdot 3xx$ $2\cdot 5xxx$ $-1\cdot 2^3x$

$2x$

b) $18y^2$, $-63y^3$, & $27y^4$

$3^2\cdot 2yy$ $-1\cdot 7\cdot 3^2yyy$ 3^3yyyy

3^2y^2 $9y^2$

c) a^3b^2, a^5b, & a^6b^2

a^3b

Practice 3: Find the GCF of each list of terms.

a) $5y^4$, $15y^2$, & $-20y^3$

$5yyyy$ $3\cdot 5yy$ $-1\cdot 2^2\cdot 5yyy$

$5y^2$

b) $4x^2$, x^3, & $3x^8$

2^2xx xxx $3xxxxxxxx$

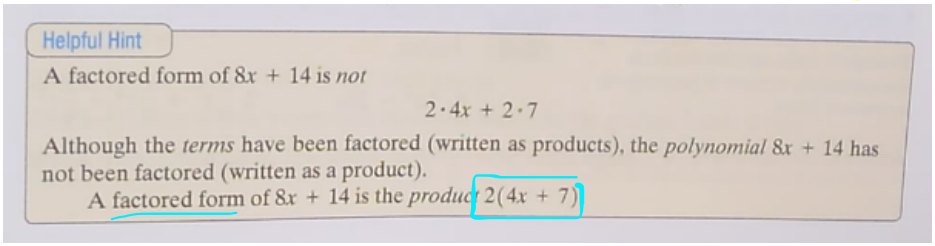
x^2

c) a^4b^2, a^3b^5, & a^2b^3

a^2b^2

OBJECTIVE 3: Factoring Out the Greatest Common Factor

First step is to factor out the GCF.



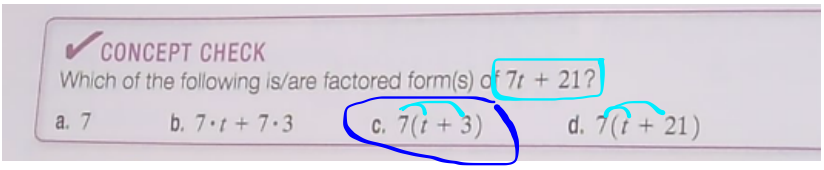
Helpful Hint

A factored form of $8x + 14$ is *not*

$$2 \cdot 4x + 2 \cdot 7$$

Although the *terms* have been factored (written as products), the *polynomial* $8x + 14$ has not been factored (written as a product).

A factored form of $8x + 14$ is the *product* $2(4x + 7)$



CONCEPT CHECK

Which of the following is/are factored form(s) of $7t + 21$?

a. 7 b. $7 \cdot t + 7 \cdot 3$ c. $7(t + 3)$ d. $7(t + 21)$

$7t + 3 \cdot 7$

$7(t + 3)$

Example 4: Factor each polynomial by factoring out the GCF.

a) $6t + 18$ $\quad 6t + 18^3$ $\quad 2 \cdot 3 \cdot t + 3^2 \cdot 2$ $\quad 2 \cdot 3 = 6$ $\quad \boxed{6(t+3)}$

b) $y^5 - y^7$ $\quad \frac{y^5}{y^5} = 1$ $\quad \frac{y^7}{y^5}$ $\quad y^{7-5}$ $\quad \boxed{y^5(1-y^2)}$

Practice 4:

a) $4t + 12$ $\quad 2^2 t + 2^2 \cdot 3$ $\quad \boxed{2^2}$ $\quad 2^2(t+3)$ $\quad \boxed{4(t+3)}$

b) $y^8 + y^4$ $\quad \frac{y^8}{y^4}$ $\quad y^{8-4}$ $\quad y^4$ $\quad \frac{y^4}{y^4} = 1$ $\quad \boxed{y^4(y^4+1)}$

Example 5: Factor: $-9a^5 + 18a^2 - 3a$

$-1 \cdot 3^2 aaaaa$ $\quad 3^2 \cdot 2aa$ $\quad -1 \cdot 3a$ $\quad 3a$

$\boxed{3a(-3a^4 + 6a - 1)}$

Practice 5: $-8b^6 + 16b^4 - 8b^2$

$-1 \cdot 2^3$ $\quad bbbbbb$ $\quad 2^4$ $\quad bbbb$ $\quad -1 \cdot 2^3$ $\quad bb$

$2^3 b^2$

$\boxed{8b^2(-b^4 + 2b^2 - 1)}$

Examples 6 - 8: Factor.

6) $6a^4 - 12a$

$6a(a^3 - 2)$

7) $\frac{3}{7}x^4 + \frac{1}{7}x^3 - \frac{5}{7}x^2$

$\frac{1}{7}x^2(3x^2 + x - 5)$

8) $15p^2q^4 + 20p^3q^5 + 5p^3q^3$

$5p^2q^3(3q + 4pq^2 + p)$

Practices 6 - 8: Factor.

6) $5x^4 - 20x$

$5x(x^3 - 4)$

7) $\frac{5}{9}z^5 + \frac{1}{9}z^4 - \frac{2}{9}z^3$

$\frac{1}{9}z^3(5z^2 + z - 2)$

8) $8a^2b^4 - 20a^3b^3 + 12ab^3$

$4ab^3(2ab - 5a^2 + 3)$

6.1 DAY ONE Assignment:

Pg. 385: 1 - 53 (o)