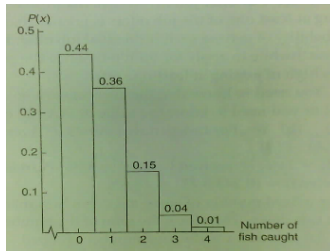


6.2 Binomial Probabilities with work

HW: p. 248: 1, 3, 5, 7, 13

1.
 - a) discrete
 - b) continuous
 - c) continuous
 - d) discrete
 - e) continuous
3. a) Yes. b) No, probabilities total to more than 1.
5. No. Even though the outcomes in the sample space are the same, the individual probabilities may differ in a way that produces the same μ but a different standard deviation.
7. Expected value = 0.9. $\sigma \approx 0.6245$
13. a) Fishing Trout in Paiute Indian Nation, Pyramid Lake, Nevada b) 0.56 c) 0.20 d) 0.82 e) 0.899



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6.2 Binomial Probabilities

Essential Question:

How is a binomial probability distribution applicable to the real-world?

FOCUS POINTS:

- List the defining features of a binomial experiment.
- Compute binomial probabilities using the formula
$$P(r) = C_{n,r} p^r q^{n-r}$$
- Use the binomial table to find $P(r)$.
- Use the binomial probability distribution to solve real-world applications.

Nov 29-9:52 AM

FEATURES OF A BINOMIAL EXPERIMENT

1. There is a **fixed number of trials**. We denote this number by the letter **n**.
2. The n trials are **independent** and repeated under identical conditions.
3. Each trial has only **two** outcomes: **success**, denoted by S, and **failure**, denoted by F.
4. For each individual trial, the **probability of success is the same**. We denote the probability of success by **p** and that of failure by **q**. Because each trial results in either success or failure, **p + q = 1** and **q = 1 - p**.
5. The central problem of a binomial experiment is to find the **probability of r successes out of n trials**.

n, p, q, r

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Example 1: TV Game Show

Wheel of Fortune is a roulette wheel with 36 slots, one is gold. If the ball lands in the gold slot the contestant wins \$50,000. No other slot pays. What is the probability that the game show will have to pay the fortune to **three** contestants out of 100?

Go through the steps 1 - 5.

1. $n = 100$
 2. independent
 3. S = gold
F = not gold
 4. $p = \frac{1}{36}$ $q = \frac{35}{36}$
 5. $r = 3$
- $n = 100$ $p = \frac{1}{36}$
 $r = 3$ $q = \frac{35}{36}$

- Full
1. $n = 100$
 2. trials are independent
 3. Gold = success (S), Gold^c = failure (F)
 4. $P(S) = \frac{1}{36}$. So, $q = 1 - p = 1 - \frac{1}{36} = \frac{35}{36}$
 5. 3 successes out of 100, so $r = 3$. The probability of the game show paying is .23.
- $$P(r) = C_{n,r} p^r q^{n-r} = {}_{100}C_3 \left(\frac{1}{36}\right)^3 \left(\frac{35}{36}\right)^{100} \approx .225$$

$$P(3) = \frac{100!}{3!(100-3)!} \left(\frac{1}{36}\right)^3 \left(\frac{35}{36}\right)^{97}$$

≈ 225

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FORMULA FOR THE BINOMIAL PROBABILITY DISTRIBUTION

$$P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r} = C_{n,r} p^r q^{n-r}$$

where n = number of binomial trials

p = probability of success on each trial

q = 1 - p = probability of failure on each trial

r = random variable representing the number of successes out of n trials ($0 \leq r \leq n$)

! = factorial notation. $1! = 1$ and $0! = 1$

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Example 2: Internet Privacy

Privacy is a concern for many users of the Internet. One survey showed that 59% of Internet users are somewhat concerned about the confidentiality of their email. Based on this information, what is the probability that for a random sample of 10 Internet users, 6 are concerned about the privacy of their e-mail?

Pull

$n = 10$
 $r = 6$
 $p = .59$
 $q = .41$

$P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r} = C_{n,r} p^r q^{n-r}$
 $P(6) = \frac{10!}{6!(4)!} (.59)^6 (.41)^4$
 $= (210)(.042)(.028)$
 $\approx .24694$

		210
.59 ⁶		
.41 ⁴		
210*.042*.028		
		.24696

.247

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6.2 Binomial Probabilities with work

USING A BINOMIAL DISTRIBUTION TABLE

Numbers in the table represent $P(X=x)$ for a binomial distribution with n trials and probability of success p .

Binomial probabilities:		0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9
0	0.990	0.890	0.790	0.690	0.590	0.500	0.400	0.300	0.250	0.200	0.100	0.000
1	0.100	0.200	0.250	0.300	0.400	0.500	0.500	0.700	0.750	0.800	0.900	0.990
2	0.810	0.640	0.563	0.490	0.360	0.210	0.100	0.090	0.063	0.040	0.010	0.000
3	0.180	0.320	0.375	0.420	0.480	0.500	0.480	0.420	0.375	0.320	0.180	0.010
4	0.010	0.040	0.063	0.090	0.160	0.210	0.360	0.490	0.563	0.640	0.810	0.990
5	0.729	0.512	0.422	0.343	0.216	0.105	0.064	0.027	0.016	0.008	0.000	0.000
6	0.243	0.384	0.422	0.441	0.432	0.375	0.288	0.189	0.141	0.096	0.027	0.000
7	0.027	0.096	0.141	0.189	0.288	0.375	0.432	0.441	0.422	0.384	0.243	0.027
8	0.001	0.008	0.016	0.027	0.064	0.105	0.160	0.216	0.288	0.375	0.422	0.441
9	0.656	0.410	0.316	0.240	0.130	0.053	0.026	0.008	0.004	0.002	0.000	0.000
10	0.292	0.410	0.422	0.412	0.346	0.250	0.154	0.076	0.047	0.026	0.004	0.000
11	0.049	0.154	0.211	0.265	0.346	0.375	0.346	0.265	0.211	0.154	0.049	0.000
12	0.004	0.026	0.047	0.076	0.154	0.250	0.346	0.412	0.422	0.410	0.292	0.049
13	0.000	0.002	0.004	0.008	0.026	0.053	0.130	0.240	0.316	0.410	0.566	0.729
14	0.590	0.328	0.237	0.168	0.078	0.031	0.010	0.002	0.001	0.000	0.000	0.000
15	0.328	0.410	0.396	0.360	0.259	0.166	0.077	0.028	0.015	0.006	0.000	0.000
16	0.073	0.205	0.264	0.309	0.346	0.313	0.230	0.132	0.088	0.051	0.008	0.000
17	0.008	0.051	0.088	0.132	0.230	0.313	0.346	0.309	0.264	0.205	0.073	0.008
18	0.000	0.006	0.015	0.028	0.077	0.156	0.259	0.360	0.396	0.410	0.328	0.073
19	0.000	0.000	0.001	0.002	0.010	0.031	0.078	0.168	0.237	0.328	0.396	0.410
20	0.531	0.262	0.178	0.118	0.047	0.016	0.004	0.001	0.000	0.000	0.000	0.000
21	0.354	0.393	0.356	0.303	0.187	0.084	0.037	0.010	0.004	0.002	0.000	0.000
22	0.098	0.246	0.297	0.324	0.311	0.234	0.138	0.060	0.033	0.015	0.001	0.000
23	0.015	0.082	0.132	0.185	0.276	0.313	0.276	0.185	0.132	0.082	0.015	0.000
24	0.001	0.015	0.033	0.060	0.138	0.234	0.311	0.324	0.297	0.246	0.098	0.015
25	0.000	0.002	0.004	0.010	0.037	0.084	0.168	0.237	0.328	0.396	0.354	0.262
26	0.000	0.000	0.000	0.001	0.004	0.016	0.047	0.118	0.178	0.262	0.354	0.410
27	0.478	0.210	0.133	0.082	0.028	0.008	0.002	0.000	0.000	0.000	0.000	0.000
28	0.372	0.367	0.311	0.247	0.131	0.055	0.017	0.004	0.001	0.000	0.000	0.000
29	0.124	0.275	0.311	0.318	0.261	0.164	0.077	0.025	0.012	0.004	0.000	0.000
30	0.023	0.115	0.173	0.227	0.290	0.373	0.194	0.097	0.058	0.029	0.003	0.000
31	0.003	0.029	0.058	0.097	0.194	0.273	0.290	0.227	0.173	0.115	0.023	0.000
32	0.000	0.004	0.012	0.025	0.077	0.164	0.261	0.318	0.311	0.275	0.124	0.023
33	0.000	0.000	0.001	0.004	0.017	0.055	0.131	0.247	0.311	0.367	0.372	0.210
34	0.000	0.000	0.000	0.000	0.002	0.008	0.028	0.082	0.133	0.210	0.478	0.531

$n = 6, p = 0.50, r/x = 4$ of less

$$P(0) + P(1) + P(2) + P(3) + P(4)$$

$$1 - [P(5) + P(6)]$$

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Example 3: Tomatoes

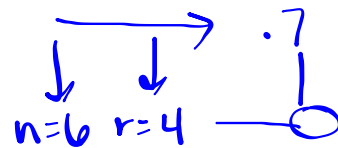
A biologist is studying a new hybrid tomato. It is known that the seeds of this hybrid tomato have probability 0.70 of germinating. The biologist plants six seeds.

a) What is the probability that exactly four seeds will germinate?

$$n = 6 \quad p = .7 \quad q = .3 \quad r = 4$$

$$P(4) = .324$$

Use the table.



b) What is the probability of at least four seeds will germinate?

$$n = 6 \quad p = .7 \quad q = .3 \quad r = 4, 5, 6$$

Use the table.

$$P(r \geq 4) = P(4) + P(5) + P(6)$$


$$= .324 + .303 + .118$$

$$= .745$$

x	0.1	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9
0	0.531	0.262	0.178	0.118	0.047	0.016	0.004	0.001	0.000	0.000
1	0.354	0.393	0.356	0.303	0.187	0.094	0.037	0.010	0.004	0.002
2	0.098	0.246	0.297	0.324	0.311	0.234	0.138	0.060	0.033	0.015
3	0.015	0.082	0.132	0.185	0.276	0.313	0.276	0.185	0.132	0.082
4	0.001	0.015	0.033	0.060	0.138	0.234	0.311	0.324	0.297	0.246
5	0.000	0.002	0.004	0.010	0.037	0.094	0.187	0.303	0.356	0.393
6	0.000	0.000	0.000	0.001	0.004	0.016	0.047	0.118	0.178	0.262

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6.2 Binomial Probabilities with work



Using the calculator:

binompdf: r = #

binomcdf: r < > < > #

```
binomcdf
  trials:6
  p: .7
  x value:4
  Paste
binomcdf(6,.7,4)
.324135
```

binom_df(n, p, r)

Summary: binompdf vs. binomcdf commands

Here are some useful applications of the binomcdf and binompdf commands:

- To find $P(x = k)$, use binompdf (n, p, k)
- To find $P(x \leq k)$, use binomcdf (n, p, k)
- To find $P(x < k)$, use binomcdf (n, p, k-1)
- To find $P(x > k)$, use 1-binomcdf (n, p, k)
- To find $P(x \geq k)$, use 1-binomcdf (n, p, k-1)

Note: k refers to some number of successes between 0 and n.

```
binompdf(6,.7,4)
.324135
binomcdf(6,.7,4)
.579825
```

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HW: p. 264: 1, 3, 5, 7, 11, 15

1. The random variable measures the number of successes out of n trials. This text uses the letter r for the random variable.
3. Two outcomes, success or failure.
5. Any monitor failure might endanger patient safety, so you should be concerned about the probability of at least one failure, not just exactly one failure.
7. a) No. A binomial probability model applies to only two outcomes per trial.
b) Yes. Assign outcome A to "success" and outcomes B and C to "failure." $p = 0.40$
11. a) 0.082
b) 0.918
15. $n = 3$; $p = 0.5$.
a) 0.125 b) 0.375 c) 0.500 d) 0.125

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