6.2 Radicals & Rational Exponents DAY ONE

Essential Question: How can you write and evaluate an nth root of a number?

The inverse of squaring is a square root:

$5^2 = 25$ $\quad \sqrt[2]{25} = \sqrt{25} = \sqrt{5^{\cancel{2}}} = \boxed{5}$

$9^2 = 81$ $\quad \sqrt[2]{81} = \sqrt{81} = \sqrt{9^{\cancel{2}}} = \boxed{9}$

$\sqrt{1} = 1 \quad \sqrt{0} = 0 \quad \sqrt{0.5} \approx 0.7071068...$

1, 4, 9, 16, 25, etc have roots that are whole numbers. Because of this: 1, 4, 9, 16, 25, etc are called perfect square numbers because a square with their area has sides that are whole or perfect numbers.

$\sqrt{1} = \sqrt{1^2} = 1$ $\quad \sqrt{5} \approx 2.236$

$\sqrt{2} \approx 1.414$ $\quad \sqrt{6} \approx 2.449$

$\sqrt{3} \approx 1.732$ $\quad \sqrt{7} \approx 2.646$

$\sqrt{4} = \sqrt{2^2} = 2$ $\quad \sqrt{8} \approx 2.828$

$\sqrt{2}$ 1.414213562
$\sqrt{3}$ 1.732050808
$\sqrt{5}$ 2.236067977
$\sqrt{6}$ 2.449489743
$\sqrt{7}$ 2.645751311
$\sqrt{8}$ 2.828427125

The root of 2, 3, 5, 6, 7, 8, etc are decimal numbers that are non-repeating and non-terminating. There is no fraction (or ratio) that equals these numbers, so they are called irrational numbers.

Examples: Use the Pythagorean Theorem to find the hypotenuse "c" of a right triangle with the given leg lengths "a" and "b". Round your answer to the nearest thousandth.

$a^2+b^2=c^2$

a) a = 3, b = 3

$(3)^2+(3)^2=c^2$

$9+9=c^2$

$\sqrt{18}=\sqrt{c^2}$

$\boxed{c \approx 4.243}$

b) a = 5, b = 7

$(5)^2+(7)^2=c^2$

$25+49=c^2$

$\sqrt{74}=\sqrt{c^2}$

$\boxed{c \approx 8.602}$

c) a = 2, b = 1

$(2)^2+(1)^2=c^2$

$4+1=c^2$

$\sqrt{5}=\sqrt{c^2}$

$\boxed{c \approx 2.236}$

YOUR TURN: Use the Pythagorean Theorem to find the hypotenuse "c" of a right triangle with the given leg lengths "a" and "b". Round your answer to the nearest thousandth.

1) a = 9, b = 2

$(9)^2+(2)^2=c^2$

$81+4=c^2$

$\sqrt{85}=\sqrt{c^2}$

$\boxed{c \approx 9.220}$

b) a = 4, b = 5

$(4)^2+(5)^2=c^2$

$16+25=c^2$

$\sqrt{41}=\sqrt{c^2}$

$\boxed{c \approx 6.403}$

c) a = 5, b = 2

$5^2+2^2=c^2$

$25+4=c^2$

$\sqrt{29}=\sqrt{c^2}$

$\boxed{c \approx 5.385}$

4.242640687

$\sqrt{74}$ 8.602325267

$\sqrt{5}$ 2.236067977

9.219544457

$\sqrt{41}$ 6.403124237

$\sqrt{29}$ 5.385164807

Vocabulary Terms:

radical sign: also called a ____ root, ie: square root or nth root

$$\sqrt{}$$

radicand: the number under the radical sign

$$\sqrt{radicand}$$

index: the root you are taking (it is not always a 2 or square root), also the number of the same number required to come out of the radicand and multiply in front

$$\sqrt{} = \sqrt[2]{} = \sqrt[index]{}$$

RECALL!!!!

When you cube a number ...

$$2^3 = 2 \bullet 2 \bullet 2 = 8$$

To "undo" cubing a number, take the cube root of the number.

$$\sqrt[3]{8} = \sqrt[3]{2^3} = \sqrt[3]{2 \bullet 2 \bullet 2} = 2$$

How do you write square root with an exponent?

Cube root?

4^{th} root?

n^{th} root?

What you will learn...

- Find n^{th} roots.
- Evaluate expressions with rational exponents.
- Solve real-life problems involving rational exponents.

Core Vocabulary:

n^{th} root

radical

index of a radical

square root

Finding n^{th} Roots

You can extend the concept of a square root to other types of roots.

ie: 2 is the cube root of 8 because $2^3 = 8$, and 3 is the fourth root of 81 because $3^4 = 81$.

In general, for an integer n greater than 1, if $b^n = a$ then b is an n^{th} root of a. An n^{th} root of a is written as $\sqrt[n]{a}$, where the expression $\sqrt[n]{a}$ is called a radical and n is the index of the radical.

You can also write an n^{th} root of a as a power of a. If you assume the Power of a Power Property applies to rational exponents, then the following is true.

$$(a^{\frac{1}{2}})^2 = a^{(\frac{1}{2})2} = a^1 = a$$

$$(a^{\frac{1}{3}})^3 = a^{(\frac{1}{3})3} = a^1 = a$$

$$(a^{\frac{1}{4}})^4 = a^{(\frac{1}{4})4} = a^1 = a$$

Because $a^{\frac{1}{2}}$ is a number whose square is a, you can write $\sqrt{a} = a^{\frac{1}{2}}$. Similarly, $\sqrt[3]{a} = a^{\frac{1}{3}}$ and $\sqrt[4]{a} = a^{\frac{1}{4}}$. In general, $\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$ for any integer n greater than 1.

CORE CONCEPT

Real *n*th Roots of a

Let *n* be an integer greater than 1, and let a be a real number.

- If *n* is odd, then a has a one real *n*th root: $\sqrt[n]{a} = a^{\frac{1}{n}}$
- If *n* is even and a > 0, then a has two real *n*th roots: $\pm\sqrt[n]{a} = \pm a^{\frac{1}{n}}$
- If *n* is even and a = 0, then a has one real *n*th root: $\sqrt[n]{0} = 0$
-
- If *n* is even and a < 0, then a has no real *n*th roots.

neg

The *n*th roots of a number may be real numbers or *imaginary numbers*. You will study imaginary numbers in Alg 2.

Find the indicated real *n*th root(s) of a.

a) n = 3, a = -27

index radicand

$\sqrt[3]{-27} = \sqrt[3]{\underbrace{-1\cdot-1\cdot-1}_{-1}\cdot\underbrace{3\cdot3\cdot3}_{3}}$

$\boxed{-3}$

b) n = 4, a = 16

$\sqrt[4]{16}$ < 4<2·2, 4<2·2

$\sqrt[4]{2^4} = \sqrt[4]{\underbrace{2\cdot2\cdot2\cdot2}_{2}}$

$\boxed{2}$

Your turn:

1) n = 3, a = 64

$\sqrt[3]{64}$

$\sqrt[3]{4^3} = \sqrt[3]{\underbrace{4\cdot4\cdot4}_{4}}$

$\boxed{4}$

2) n = 4, a = 81

$\sqrt[4]{81}$

$\sqrt[4]{3^4} = \sqrt[4]{\underbrace{3\cdot3\cdot3\cdot3}_{3}}$

$\boxed{3}$

Evaluating Expressions with Rational Exponents

Remember that the radical $\sqrt{a}$ indicates the positive square root of a. Similarly, an *n*th root of a, $\sqrt[n]{a}$, with an *even* index indicates the positive *n*th root of a.

Example: Evaluate each expression.

a) $\sqrt[3]{-8}$ $= \sqrt{\underbrace{(-1)^3}_{-1}\underbrace{(2)^3}_{2}}$

$= (-1)(2)$

$= \boxed{-2}$

b) $16^{\frac{1}{4}}$ $= \sqrt[4]{16} = \sqrt[4]{2^4}$

$= \sqrt[4]{\underbrace{2\cdot2\cdot2\cdot2}_{2}} = \boxed{2}$

Your Turn:

1) $-\sqrt[3]{8}$ $= -\sqrt[3]{2^3}$

$= -\sqrt[3]{\underbrace{2\cdot2\cdot2}_{2}}$

$= \boxed{-2}$

2) $(-16)^{\frac{1}{4}}$

$\sqrt[4]{-16}$

even "n"
neg. "a"

not possible

$\boxed{\text{nonreal}}$

6.2 DAY ONE Assignment

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