6.3 Exponential Functions

Essential Question:

What are some of the characteristics of the graph of an exponential function?

What You Will Learn:

- Identify and evaluate exponential functions.
- Graph exponential functions.
- Solve real-life problems involving exponential functions.

Core Vocabulary

exponential function

independent variable

dependent variable

parent function

Exponential function: the variable is in the exponent

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Identifying & Evaluating Exponential Functions

An exponential function is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor, which means consecutive y-values form a constant ratio.

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EXAMPLE 1 **Identifying Functions**

Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	1	2	3
y	2	4	6	8

linear

b.

x	0	1	2	3
y	4	8	16	32

exponential

SOLUTION

a.

x	0	1	2	3
y	2	4	6	8

+1 +1 +1 ; +2 +2 +2

▶ As x increases by 1, y increases by 2. The rate of change is constant. So, the function is linear.

b.

x	0	1	2	3
y	4	8	16	32

+1 +1 +1 ; ×2 ×2 ×2

▶ As x increases by 1, y is multiplied by 2. So, the function is exponential.

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More Practice:

PEMDAS

Evaluate each function for the given value of x.

a. $y = -2(5)^x$; $x = 3$

$= -2(5)^3$

$= -2(5 \cdot 5 \cdot 5)$

$= -2(125) = -250$

$-2(5)^3$ → -250

$3(0.5)^{-2}$ → 12

b. $y = 3(0.5)^x$; $x = -2$

$= 3\left(\frac{1}{2}\right)^{-2}$

$= 3\left(\frac{2}{1}\right)^2$

$= 3(4) = 12$

Your Turn:

Evaluate each function for the given value of x.

a. $y = -3\ (4)^x$; $x = 2$

$= -3\ (4)^2$

$= -3(4 \cdot 4)$

$= -3(16)$

$= -48$

$-3(4)^2$ → -48

$2(0.25)^2$ → $.125$

Ans▸Frac → $\frac{1}{8}$

b. $y = 2\ (0.25)^x$; $x = 2$

$= 2\left(\frac{1}{4}\right)^2$

$= 2\left(\frac{1}{4} \cdot \frac{1}{4}\right)$

$= 2\left(\frac{1}{16}\right) = \frac{2}{16} = \frac{1}{8}$

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Steps:

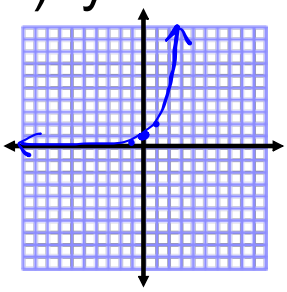
1) Make a t-table of values.

2) Plot the ordered pairs from your table.

3) Draw a smooth curve through the points.

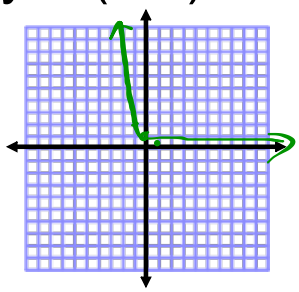
Graph:

1) $y = 2^x$



x	2^x	y
-1	2^{-1}	$\frac{1}{2}$
0	2^0	1
1	2^1	2

2) $y = (1/2)^x$



x	$\left(\frac{1}{2}\right)^x$	y
-1	$\left(\frac{1}{2}\right)^{-1}$	2
0	$\left(\frac{1}{2}\right)^0$	1
1	$\left(\frac{1}{2}\right)^1$	$\frac{1}{2}$

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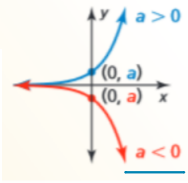
Graphing Exponential Functions

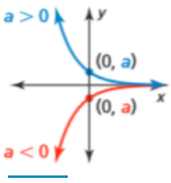
The graph of a function $y = ab^x$ is a vertical stretch or shrink by a factor of $|b|$ of the graph of the parent function $y = b^x$. When $a < 0$, the graph is also reflected over the x-axis. The y-intercept of the graph of $y = ab^x$ is a.

Core Concept

Graphing $y = ab^x$ When $b > 1$



Graphing $y = ab^x$ When $0 < b < 1$



STUDY TIP

The graph of $y = ab^x$ approaches the x-axis but never intersects it.

$a = -4$

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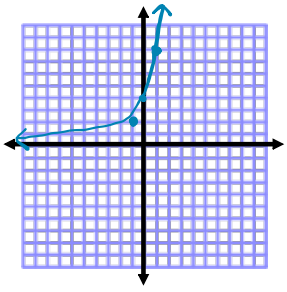
Example: Graphing when b > 1

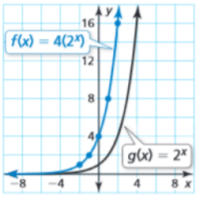
Graph $f(x) = 4(2)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of f.

Make a table, plot the points, connect the coordinates.



x	$4(2)^x$	y
-1	$4(2)^{-1}$ $4(\frac{1}{2})$	2
0	$4(2)^0$ $4(1)$	4
1	$4(2)^1$	8

The parent function is $g(x) = 2^x$. The graph of f is a vertical stretch by a factor of 4 on the graph of g. The y-intercept of the graph of f, (0, 4), is above the y-intercept of the graph of g, (0, 1). From the graph of f, you can see the domain is all real numbers and the range is y >0.



x	f(x)
-2	1
-1	2
0	4
1	8
2	16

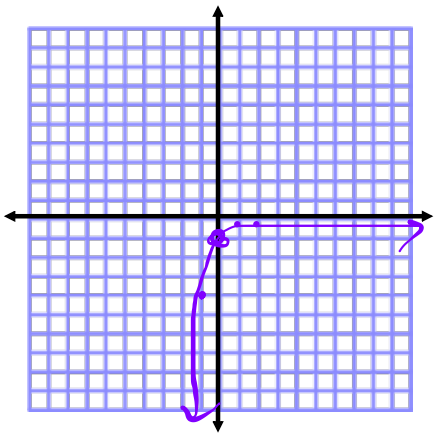
Example

Graph $f(x) = -(0.25)^x$

D: ℝ
R: $y < 0$
VS $\frac{1}{4}$
R_x

Compare the graph to the graph of the parent function. Describe the domain and range of f.

x	$-(0.25)^x$	f(x) or y
−2	$-(\frac{1}{4})^{-2} = -(4)^2$	−16
−1	$-(\frac{1}{4})^{-1} = -(4)^1$	−4
0	$-(\frac{1}{4})^0$	−1
1	$-(\frac{1}{4})^1$	$-\frac{1}{4}$
2	$-(\frac{1}{4})^2 = -\frac{1}{16}$	$-\frac{1}{16}$



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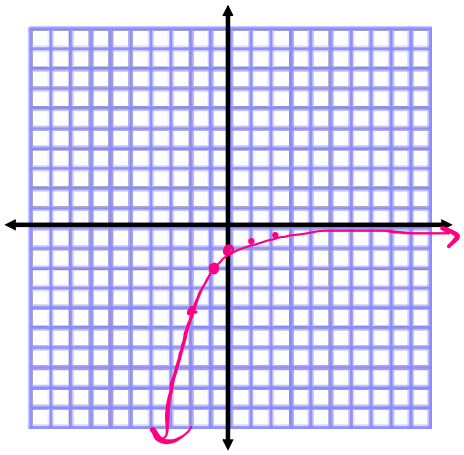
Your Turn:

$$f(x) = -\left(\frac{1}{2}\right)^x$$

VS $\frac{1}{2}$
R_x
D: ℝ
R: $y < 0$

Compare the graph to the graph of the parent function. Describe the domain and range of f.

x	-(1/2)x	f(x)
−2	$-(\frac{1}{2})^{-2} = -(2)^2$	−4
−1	$-(\frac{1}{2})^{-1} = -(2)$	−2
0	$-(\frac{1}{2})^0$	−1
1	$-(\frac{1}{2})^1$	$-\frac{1}{2}$
2	$-(\frac{1}{2})^2$	$-\frac{1}{4}$



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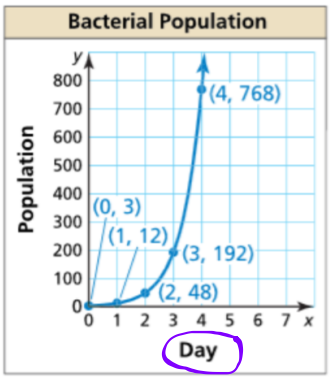
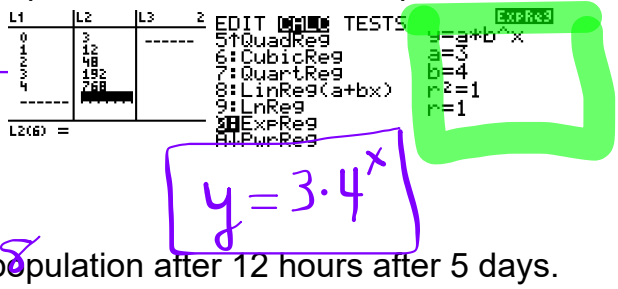
Real-World Application

The graph represents a bacterial population y after x days.



a) Write an exponential function that represents the population.

x	y
0	3
1	12
2	48
3	192
4	768

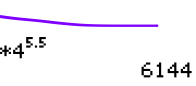


$y = 3 \cdot 4^x$

b) Find the population after 12 hours after 5 days.

$x = 5.5$

$y = 3 \cdot 4^{5.5}$



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Assignment:

pg. 310: 5 - 12, 21 - 24, 27, 28, 50, 51