6.5 Factoring Binomials

OBJECTIVE 1: Factoring the Difference of Two Squares

Reminder of special products from Chapter 5.

$$\boxed{(a + b)(a - b) = (a^2 - b^2)}$$

This binomial is called a difference of squares.

Factoring the Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Factoring the Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Helpful Hint

Since multiplication is commutative, remember that the order of factors does not matter. In other words,

$$a^2 - b^2 = (a + b)(a - b) \text{ or } (a - b)(a + b)$$

Example 1: Factor $x^2 - 25$

$a^2 = x^2$ $b^2 = 25$

$a = x$ $b = 5$

$\boxed{(x+5)(x-5)}$

Practice 1: $x^2 - 81$

$a = x$ $b = 9$ $\boxed{(x+9)(x-9)}$

Example 2: Factor each difference of squares.

a) $4x^2 - 1$

$2x \quad 1$

$\boxed{(2x+1)(2x-1)}$

b) $25a^2 - 9b^2$

$5a \quad 3b$

$\boxed{(5a+3b)(5a-3b)}$

c) $y^2 - \frac{4}{9}$

$y \quad \frac{2}{3}$

$\boxed{(y+\frac{2}{3})(y-\frac{2}{3})}$

Practice 2:

a) $9x^2 - 1$

$3x \quad 1$

$\boxed{(3x+1)(3x-1)}$

b) $36a^2 - 49b^2$

$6a \quad 7b$

$\boxed{(6a-7b)(6a+7b)}$

c) $p^2 - \frac{25}{36}$

$p \quad \frac{5}{6}$

$\boxed{(p+\frac{5}{6})(p-\frac{5}{6})}$

Example 3: Factor $x^4 - y^6$

$x^2 \quad y^3$

$\boxed{(x^2+y^3)(x^2-y^3)}$

Practice 3: $p^4 - q^{10}$

$p^2 \quad q^5$

$\boxed{(p^2+q^5)(p^2-q^5)}$

Example 4: Factor each binomial.

a) $\sqrt{y^4} - \sqrt{16}$

$y^2 \quad 4$

$(y^2+4)(\sqrt{y^2}-\sqrt{4})$

$y \quad 2$

$\boxed{(y^2+4)(y+2)(y-2)}$

b) $x^2 + 4$

$\boxed{\text{non-factorable}}$

Practice 4:

a) $\sqrt{z^4} - \sqrt{81}$

$z^2 \quad 9$

$(z^2+9)(\sqrt{z^2}-\sqrt{9})$

$z \quad 3$

$\boxed{(z^2+9)(z+3)(z-3)}$

b) $m^2 + 49$

$\boxed{\text{non-factorable}}$

Helpful Hint

When factoring, don't forget:

- See whether the terms have a greatest common factor (GCF) (other than 1) that can be factored out.
- Other than the GCF, the **sum** of two squares cannot be factored using real numbers.
- Factor completely. Always check to see whether any factors can be factored further.

Example 5 & 6: Factor each binomial.

5. $4x^3 - 49x$

$x(\sqrt{4x^2}-\sqrt{49})$

$2x \quad 7$

$\boxed{x(2x+7)(2x-7)}$

6. $162x^2 - 2$

$2(\sqrt{81x^2}-\sqrt{1})$

$9x \quad 1$

$\boxed{2(9x+1)(9x-1)}$

Practice 5 & 6: Factor each binomial.

5. $36y^3 - 25y$

$y(\sqrt{36y^2} - \sqrt{25})$

$6y \quad 5$

$\boxed{y(6y+5)(6y-5)}$

6. $80y^2 - 5$

$5(\sqrt{16y^2} - \sqrt{1})$

$4y \quad 1$

$\boxed{5(4x+1)(4y-1)}$

Example 7: Factor $-49x^2 + 16$

$-1(\sqrt{49x^2} - \sqrt{16})$

$7x \quad 4$

$\boxed{-(7x+4)(7x-4)}$

Practice 7: $-9x^2 + 100$

$-(\sqrt{9x^2} - \sqrt{100})$

$3x \quad 10$

$\boxed{-1(3x+10)(3x-10)}$

OBJECTIVE 2: Factoring the Sum or Difference of Two Cubes

Sum of squares usually does not factor, but the sum and difference of cubes does work. The pattern leads to these two different formulas which I refer to as SOAP to help remember the signs of the formulas because the letters are the same in both!

Factoring the Sum or Difference of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Remember "factor" means to "write out as products"

Example 8: Factor $\sqrt[3]{x^3} + \sqrt[3]{8}$ SOAP

$(a+b)(a^2 - ab + b^2)$

$a = x \quad b = 2$

$$\boxed{(x+2)(x^2 - 2x + 4)}$$

Practice 8: $\sqrt[3]{x^3} + \sqrt[3]{64}$

$a = x \quad b = 4$

$$\boxed{(x+4)(x^2 - 4x + 16)}$$

SOAP

Helpful Hint

When factoring sums or differences of cubes, notice the sign patterns.

same sign

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

opposite signs always positive

same sign

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

opposite signs always positive

Example 9: Factor $y^3 - 27$

$\sqrt[3]{y^3}$ $\sqrt[3]{27}$

$a = y$ $b = 3$

$(a - b)(a^2 + ab + b^2)$

$$\boxed{(y-3)(y^2+3y+9)}$$

Practice 9: $x^3 - 125$

$\sqrt[3]{x^3}$ $\sqrt[3]{125}$

$a = x$ $b = 5$

$$\boxed{(x-5)(x^2+5x+25)}$$

Example 10: Factor $64x^3 + 1$

$\sqrt[3]{64x^3} + \sqrt[3]{1}$

$4x \quad 1$

$\boxed{(4x+1)(16x^2-4x+1)}$

Practice 10: $27y^3 + 1$

$\sqrt[3]{27y^3} + \sqrt[3]{1}$

$3y \quad 1$

$\boxed{(3y+1)(9y^2-3y+1)}$

Example 11: Factor $54a^3 - 16b^3$

$2(\sqrt[3]{27a^3} - \sqrt[3]{8b^3})$

$3a \quad 2b$

$\boxed{2(3a-2b)(9a^2+6ab+4b^2)}$

Practice 11: $32x^3 - 500y^3$

$4(\sqrt[3]{8x^3} - \sqrt[3]{125y^3})$

$2x \quad 5y$

$\boxed{4(2x-5y)(4x^2+10xy+25y^2)}$

Graphing Calculator Explorations

TEACHING TIP
Some graphing calculators have a TABLE feature that allows the user to evaluate an expression for various values. Enter the expressions using the Y = key. Then use the TABLE feature.

Graphing

A graphing calculator is a convenient tool for evaluating an expression at a given replacement value. For example, let's evaluate $x^2 - 6x$ when $x = 2$. To do so, store the value 2 in the variable x and then enter and evaluate the algebraic expression.

```
2→X
                2
X²-6X
               -8
```

The value of $x^2 - 6x$ when $x = 2$ is -8. You may want to use this method for evaluating expressions as you explore the following.

We can use a graphing calculator to explore factoring patterns numerically. Use your calculator to evaluate $x^2 - 2x + 1$, $x^2 - 2x - 1$, and $(x - 1)^2$ for each value of x given in the table. What do you observe?

	$x^2 - 2x + 1$	$x^2 - 2x - 1$	$(x - 1)^2$
$x = 5$	16	14	16
$x = -3$	16	14	16
$x = 2.7$	2.89	0.89	2.89
$x = -12.1$	171.61	169.61	171.61
$x = 0$	1	−1	1

Notice in each case that $x^2 - 2x - 1 \neq (x - 1)^2$. Because for each x in the table the value of $x^2 - 2x + 1$ and the value of $(x - 1)^2$ are the same, we might guess that $x^2 - 2x + 1 = (x - 1)^2$. We can verify our guess algebraically with multiplication:

$$(x - 1)(x - 1) = x^2 - x - x + 1 = x^2 - 2x + 1$$

Use the choices below to fill in each blank. Some choices may be used more than once and some choices may not be used at all.

true	difference of two squares	sum of two cubes
false	difference of two cubes	

1. The expression $x^3 - 27$ is called a difference of two cubes.
2. The expression $x^2 - 49$ is called a difference of two squares.
3. The expression $z^3 + 1$ is called a sum of two cubes.
4. True or false: The binomial $y^2 + 9$ factors as $(y + 3)^2$. false

Write each term as a square.

5. $49x^2$ $(7x)^2$
6. $25y^4$ $(5y^2)^2$

Write each term as a cube.

7. $8y^3$ $(2y)^3$
8. x^6 $(x^2)^3$

6.5 HW Assignment

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