6.6 Geometric Sequences

Essential Question:

How can you use a geometric sequence to describe a pattern?

What You Will Learn:

- Identify geometric sequences.
- Extend & graph geometric sequences.
- Write geometric sequences as functions.

Core Vocabulary:

geometric sequence

common ratio

arithmetic sequence

common difference

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Arithmetic: $5, 8, 11, 14 \ldots$ $d = 3$

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**.

$\cdot r \quad \cdot r \quad \cdot r$

Each term is found by multiplying the previous term by the common ratio.

$= r$

$a_1 = 1$

$r = 5$

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Examples:

Decide whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

a. 120, 60, 30, 15, . . .

$\div 2 = \cdot \frac{1}{2}$

$\boxed{r = \frac{1}{2} \text{ geometric}}$

b. 2, 6, 11, 17, . . .

+4 +5

not geometric → no r
not arithmetic → no d
neither

Write the next three terms of each geometric sequence.

a. 3, 6, 12, 24, . . .

·2 ·2 ·2

$r = 2$

$\boxed{48, 96, 192}$

$24 \cdot 2 = 48$
$48 \cdot 2 = 96$
$96 \cdot 2 = 192$

b. 64, −16, 4, −1, . . .

$\div -4 \Rightarrow \cdot -\frac{1}{4}$ $\quad r = -\frac{1}{4}$

$-1 \cdot \left(-\frac{1}{4}\right) = \frac{1}{4}$
$\frac{1}{4}\left(-\frac{1}{4}\right) = -\frac{1}{16}$
$-\frac{1}{16}\left(-\frac{1}{4}\right) = \frac{1}{64}$

$\boxed{\frac{1}{4}, -\frac{1}{16}, \frac{1}{64}}$

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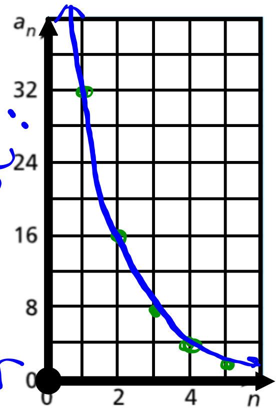
Graphing Geometric Sequences

Graph the geometric sequence 32, 16, 8, 4, 2, ... What do you notice?

Make a table and plot those points.

n	a_n
1	32
2	16
3	8
4	4
5	2

We don't connect these, but it helps visualize.



Arithmetic → linear
Straight line

Geometric → exponential growth/decay
curved graph

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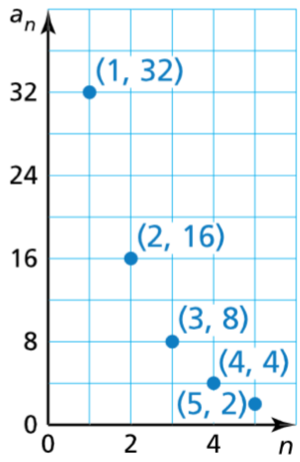
SOLUTION

Make a table. Then plot the ordered pairs (n, a_n).

Position, n	1	2	3	4	5
Term, a_n	32	16	8	4	2



▶ The points appear to lie on an exponential curve.

Writing Geometric Sequences as Functions

Because consecutive terms of a geometric sequence have a common ratio, you can use the first term a_1 and the common ratio r to write an exponential function that describes a geometric sequence. Let $a_1 = 1$ and $r = 5$.

1 5 25 125...

$r \cdot r = r^2$

$r \cdot r \cdot r = r^3$

$10^{th} \rightarrow a_1 r^9$

Position, n	Term, a_n	Written using a_1 & r	Numbers
1	first term, a_1	a_1	1
2	second term, a_2	a_1r	1 (5) = 5
3	third term, a_3	a_1r^2	$1\ (5)^2 = 25$
4	fourth term, a_4	a_1r^3	$1\ (5)^3 = 125$
...	...	...	...
n	nth term, a_n	a_1r^{n-1}	$1\ (5)^{n-1} = ?$

$$a_n = a_1(r)^{n-1}$$

Core Concept

Equation for a Geometric Sequence

Let a_n be the nth term of a geometric sequence with first term a_1 and common ration r. The nth term is given by...

STUDY TIP

Notice that the equation $a_n = a_1 r^{n-1}$ is of the form $y = ab^x$.

$$\boxed{a_n = a_1 r^{n-1}}$$

Example 4: Finding the nth Term of a Geometric Sequence

Write an equation for the nth term of the geometric sequence 2, 12, 72, 432, ... Then find $\boxed{a_{10}}$.

a_1

$r = 6$

$2(6)^{10-1}$ 20155392

$a_n = a_1 \cdot r^{n-1}$

$a_n = 2 \cdot 6^{n-1}$

$a_{10} = 2 \cdot (6)^9 = \boxed{20{,}155{,}392}$

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Write an equation for the *n*th term of the geometric sequence 2, 12, 72, 432, ... Then find a_{10}.

SOLUTION

The first term is 2, and the common ratio is 6.

$a_n = a_1 r^{n-1}$ Equation for a geometric sequence

$a_n = 2(6)^{n-1}$ Substitute 2 for a_1 and 6 for *r*.

Use the equation to find the 10th term.

Your Turn:

$a_n = a_1 \cdot r^{n-1}$

Write an equation for the nth term of the geometric sequence. Then find $\boxed{a_7}$.

1) 1, -5, 25, -125, ...

$\cdot -5 \quad \cdot -5 \quad \cdot -5$

$a_1 = 1 \quad r = -5$

$a_n = 1(-5)^{n-1}$

$a_7 = 1(-5)^6 = \boxed{15{,}625}$

1(-5)^(7-1)
15625

2) 13, 26, 52, 104, ...

$\cdot 2 \quad \cdot 2 \quad \cdot 2$

$a_1 = 13 \quad r = 2$

$a_7 = 13(2)^6 = \boxed{832}$

13(2)^6
832

3) 432, 72, 12, 2, ...

$\div 6$

$\cdot \frac{1}{6} \quad \cdot \frac{1}{6} \quad \cdot \frac{1}{6}$

$a_1 = 432 \quad r = \frac{1}{6}$

$a_n = 432\left(\frac{1}{6}\right)^{n-1}$

$a_7 = 432\left(\frac{1}{6}\right)^6 = \boxed{\frac{1}{108} \approx 0.009}$

432(1/6)^(7-1)
.0092592593
Ans▸Frac
1/108

4) 4, 10, 25, 62.5, ...

$4 \cdot ? = 10$

$10 \cdot ? = 25$

$\frac{10}{4} = 2.5$

$\frac{25}{10} = 2.5$

$\cdot 2.5 \quad \cdot 2.5 \quad \cdot 2.5$

$a_1 = 4 \quad r = 2.5$

$a_n = 4(2.5)^{n-1}$

$a_7 = 4(2.5)^{7-1}$

$\boxed{= \frac{15{,}625}{16} = 976.5625}$

4(2.5)^(7-1)
976.5625
Ans▸Frac
15625/16

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Real-Life Problems

Clicking the *zoom-out* button on a mapping website doubles the side length of the square map. After how many clicks on the *zoom-out* button is the side length of the map 640 miles?

Zoom-out clicks	1	2	3	?
Map side length (miles)	5	10	20	640

$a_1 = 5$

$r = 2$

$a_n = 640$

$a_n = a_1 (r)^{n-1}$

$a_n = 5(2)^{n-1}$

$\frac{640}{5} = \frac{\cancel{5}(2)^{n-1}}{\cancel{5}}$

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$7 = n-1$

$\boxed{8 = n}$

If I click on the map 8 times, then the side length of the map will be 640 miles.

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6.6 Assignment

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