6.6 Solving Quadratic Equations by Factoring

Quadratic Equation

A quadratic equation is one that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$.

The quadratic equation has a degree of two ALWAYS. In standard form it is $ax^2 + bx + c = 0$. These equations can be very real world, like jumping off a cliff, punting a football, and many more.

OBJECTIVE 1: Solving Quadratic Equations by Factoring

Some quadratic equations can be solved by making use of factoring and the zero factor property (ZFP).

Zero Factor Property

If a and b are real numbers and if $ab = 0$, then $a = 0$ or $b = 0$.

This property states that if the product of two numbers is 0 then at least one of the numbers must be 0.

Example 1: Solve $(x - 3)(x + 1) = 0$.

$x - 3 = 0$ $\quad$ $x + 1 = 0$

$+3 \quad +3$ $\quad$ $-1 \quad -1$

$\boxed{x = 3}$ $\quad$ $\boxed{x = -1}$

Practice 1: $(x + 4)(x - 5) = 0$

$x + 4 = 0$ $\quad$ $x - 5 = 0$

$-4 \quad -4$ $\quad$ $+5 \quad +5$

$\boxed{x = -4}$ $\quad$ $\boxed{x = 5}$

Helpful Hint

The zero factor property says that *if a product is 0, then a factor is 0.*

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

If $x(x + 5) = 0$, then $x = 0$ or $x + 5 = 0$.

If $(x + 7)(2x - 3) = 0$, then $x + 7 = 0$ or $2x - 3 = 0$.

Use this property only when the product is 0.

For example, if $a \cdot b = 8$, we do not know the value of a or b. The values may be $a = 2, b = 4$ or $a = 8, b = 1$, or any other two numbers whose product is 8.

Example 2: Solve (x - 5)(2x + 7) = 0

$x - \cancel{5} = 0$
$\quad + \cancel{5} \quad +5$
$\boxed{x = 5}$

$2x + \cancel{7} = 0$
$\quad -\cancel{7} \quad -7$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{-7}{2}$
$\boxed{x = -\frac{7}{2}}$

Practice 2: (x - 12) (4x + 3) = 0

$x - \cancel{12} = 0$
$\quad +\cancel{12} \quad +12$
$\boxed{x = 12}$

$4x + \cancel{3} = 0$
$\quad -\cancel{3} \quad -3$
$\frac{\cancel{4}x}{\cancel{4}} = -\frac{3}{4}$
$\boxed{x = -\frac{3}{4}}$

Example 3: Solve x(5x - 2) = 0

$\boxed{x = 0}$

$5x - \cancel{2} = 0$
$\quad +\cancel{2} \quad +2$
$\frac{\cancel{5}x}{\cancel{5}} = \frac{2}{5}$
$\boxed{x = \frac{2}{5}}$

Practice 3: x(7x - 6) = 0

$\boxed{x = 0}$

$7x - \cancel{6} = 0$
$\quad +\cancel{6} \quad +6$
$\frac{\cancel{7}x}{\cancel{7}} = \frac{6}{7}$
$\boxed{x = \frac{6}{7}}$

Example 4: Solve $x^2 - 9x - 22 = 0$

$$(x+2)(x-11)=0$$

$$x+\cancel{2}=0 \quad -\cancel{2} \quad -2 \qquad \boxed{x=-2}$$

$$x-\cancel{11}=0 \quad +\cancel{11} \quad +11 \qquad \boxed{x=11}$$

-22

$1 \quad -22$

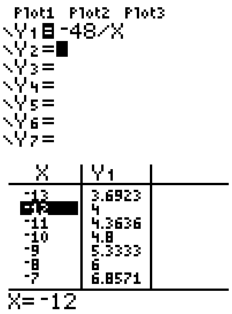
$\boxed{2 \quad -11}$

Practice 4: $x^2 - 8x - 48 = 0$

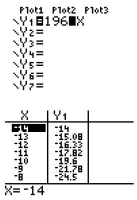
$$(x-12)(x+4)=0$$

$$x-\cancel{12}=0 \quad +\cancel{12} \quad +12 \qquad \boxed{x=12}$$

$$x+\cancel{4}=0 \quad -\cancel{4} \quad -4 \qquad \boxed{x=-4}$$



Example 5: Solve $4x^2 - 28x = -49$



$$4x^2-28x+49=0$$

$$(4x^2-14x)(-14x+49)=0$$

$$2x(2x-7)-7(2x-7)=0$$

$$(2x-7)(2x-7)=0$$

$$(2x-7)^2=0$$

$$2x-\cancel{7}=0 \quad +\cancel{7} \quad +7$$

$$\frac{\cancel{2}x}{\cancel{2}}=\frac{7}{2} \qquad \boxed{x=\frac{7}{2}}$$

a·c
$4 \cdot 49$
196
$-14 \quad -14$
-28
b

Practice 5: $9x^2 - 24x = -16$

$$9x^2-24x+16=0$$

$$(9x^2-12x)(-12x+16)=0$$

$$3x(3x-4)-4(3x-4)=0$$

$$(3x-4)(3x-4)=0$$

$$(3x-4)^2=0$$

$$3x-\cancel{4}=0 \quad +\cancel{4} \quad +4$$

$$\frac{\cancel{3}x}{\cancel{3}}=\frac{4}{3}$$

$$\boxed{x=\frac{4}{3}}$$

a·c
$9 \cdot 16$
144
$-12 \quad -12$
-24
b

To Solve Quadratic Equations by Factoring

Step 1. Write the equation in standard form so that one side of the equation is 0.

Step 2. Factor the quadratic expression completely.

Step 3. Set each factor containing a variable equal to 0.

Step 4. Solve the resulting equations.

Step 5. Check each solution in the original equation.

Example 6: Solve x(2x - 7) = 4

$2x^2 - 7x - 4 = 0$

$(2x^2 - 8x) + (x - 4) = 0$

$2x(x-4) + 1(x-4) = 0$

$(x-4)(2x+1) = 0$

a·c: 2·4, −8, +1, −7, b

$x - 4 = 0$, $+4\ +4$, $\boxed{x = 4}$

$2x + 1 = 0$, $-1\ -1$, $\frac{2x}{2} = \frac{-1}{2}$, $\boxed{x = -\frac{1}{2}}$

Practice 6: x(3x + 7) = 6

a·c: 3·−6, −18, −2, +9, 7, b

$3x^2 + 7x - 6 = 0$

$(3x^2 + 9x)(-2x - 6) = 0$

$3x(x+3) - 2(x+3) = 0$

$(3x - 2)(x + 3) = 0$

$3x - 2 = 0$, $+2\ +2$, $\frac{3x}{3} = \frac{2}{3}$, $\boxed{x = \frac{2}{3}}$

$x + 3 = 0$, $-3\ -3$, $\boxed{x = -3}$

CONCEPT CHECK

Explain the error and solve the equation correctly.

$$(x-3)(x+1) = 5$$

~~$x - 3 = 5$ or $x + 1 = 5$~~

~~$x = 8$ or $x = 4$~~

Must be = 0 before you can set the factors equal.

$x^2 - 3x + x - 3 = 5$

$x^2 - 2x - 8 = 0$ ✓

$(x-4)(x+2) = 0$

$x - 4 = 0$ $x + 2 = 0$

$\boxed{x = 4, -2}$

Example 7: Solve $-2x^2 - 4x + 30 = 0$

$\frac{-2x^2 - 4x + 30}{-2} = \frac{0}{-2}$

$\frac{\cancel{-2}x^2}{\cancel{-2}} + \frac{\cancel{-4}x}{\cancel{-2}} + \frac{\cancel{30}}{\cancel{-2}} = \frac{0}{-2}$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x+5=0 \quad x-3=0$

$\boxed{x=-5} \quad \boxed{x=3}$

$\frac{0}{K} = 0$

$\frac{N}{0} = \emptyset$

Practice 7: $-3x^2 - 6x + 72 = 0$

$\frac{-3x^2 - 6x + 72}{-3} = \frac{0}{-3}$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x+6=0 \quad x-4=0$

$\boxed{x=-6} \quad \boxed{x=4}$

OBJECTIVE 2: Solving Equations with Degree Greater Than Two by Factoring

Some equations involving polynomials of degree higher than 2 may also be solved by factoring and then applying the ZFP.

Example 8: Solve $3x^3 - 12x = 0$

$3x(x^2 - 4) = 0$

$(3x)(x+2)(x-2) = 0$

$3x = 0 \quad x+2=0 \quad x-2=0$

$\boxed{x=0} \quad \boxed{x=-2} \quad \boxed{x=2}$

Practice 8: $7x^3 - 63x = 0$

$7x(x^2-9) = 0$

$7x(x+3)(x-3) = 0$

$7x=0 \quad x+3=0 \quad x-3=0$

$\boxed{x=0} \quad \boxed{x=-3} \quad \boxed{x=3}$

Example 9: Solve $(5x - 1)(2x^2 + 15x + 18) = 0$

36, 3, 12, 15

$(5x-1)((2x^2+12x)+(3x+18))=0$

$(5x-1)[2x(x+6)+3(x+6)]=0$

$(5x-1)(2x+3)(x+6)=0$

$5x-1=0$ | $2x+3=0$ | $x+6=0$
$5x=1$ | $2x=-3$ | $\boxed{x=-6}$
$\boxed{x=\frac{1}{5}}$ | $\boxed{x=\frac{-3}{2}}$ |

Practice 9: $(3x - 2)(2x^2 - 13x + 15) = 0$

30, -13

$(3x-2)[(2x^2-10x)-(3x+15)]=0$

$(3x-2)[2x(x-5)-3(x-5)]=0$

$(3x-2)(2x-3)(x-5)=0$

$3x-2=0$ | $2x-5=0$ | $x-5=0$
$3x=2$ | $2x=5$ | $\boxed{x=5}$
$\boxed{x=\frac{2}{3}}$ | $\boxed{x=\frac{5}{2}}$ |

Example 10: Solve $2x^3 - 4x^2 - 30x = 0$

$$\frac{2x^3 - 4x^2 - 30x}{2x} = \frac{0}{2x}$$

$2x[x^2-2x-15]=0$

$2x(x-5)(x+3)=0$

$2x=0$ | $x-5=0$ | $x+3=0$
$\boxed{x=0}$ | $\boxed{x=5}$ | $\boxed{x=-3}$

Practice 10: $5x^3 + 5x^2 - 30x = 0$

$5x(x^2+x-6)=0$

$5x(x+3)(x-2)=0$

$5x=0$ | $x+3=0$ | $x-2=0$
$\boxed{x=0}$ | $\boxed{x=-3}$ | $\boxed{x=2}$

OBJECTIVE 3: Finding x-Intercepts of the Graph of a Quadratic Equation

Remember to find an x-intercept you need y to equal 0.

Example 11: Find the x-intercepts of the graph of $y = x^2 - 5x + 4$.

0

(x, 0)

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x - 4 = 0$ $x - 1 = 0$

$\boxed{x = 4}$ $\boxed{x = 1}$

$\boxed{(4,0) \text{ \& } (1,0)}$

Practice 11: $y = x^2 - 6x + 8$

$0 = x^2 - 6x + 8$

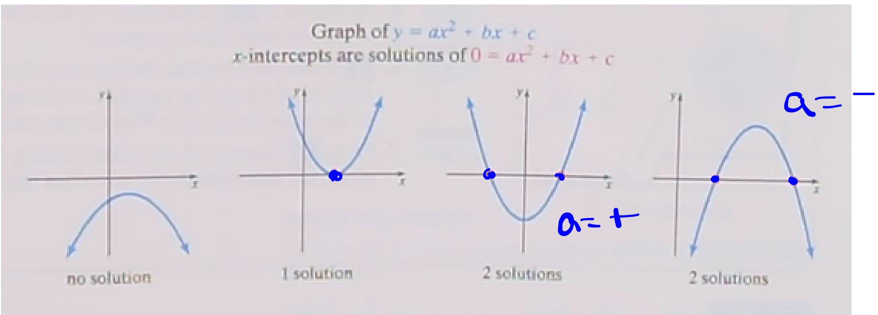
$0 = (x-4)(x-2)$

$x - 4 = 0$ $x - 2 = 0$

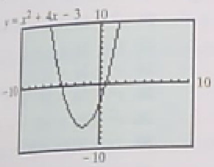
$x = 4$ $x = 2$

$\boxed{(4,0)}$ $\boxed{(2,0)}$

A quadratic equation in two variables is one that can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$. The graph is called a parabola and will open up (+ a) or down (- a) depending on the sign of a.

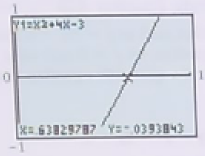


Graphing Calculator Explorations



A grapher may be used to find solutions of a quadratic equation whether the related quadratic polynomial is factorable or not. For example, let's use a grapher to approximate the solutions of $0 = x^2 + 4x - 3$. To do so, graph $y_1 = x^2 + 4x - 3$. Recall that the x-intercepts of this graph are the solutions of $0 = x^2 + 4x - 3$.

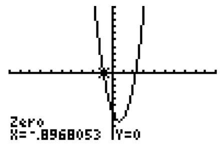
Notice that the graph appears to have an x-intercept between -5 and -4 and one between 0 and 1. Many graphers contain a TRACE feature. This feature activates a graph cursor that can be used to *trace* along a graph while the corresponding x- and y-coordinates are shown on the screen. Use the TRACE feature to confirm that x-intercepts lie between -5 and -4 and between 0 and 1. To approximate the x-intercepts to the nearest tenth, use a ROOT or a ZOOM feature on your grapher or redefine the viewing window. (A ROOT feature calculates the x-intercept. A ZOOM feature magnifies the viewing window around a specific location such as the graph cursor.) If we redefine the window to $[0, 1]$ on the x-axis and $[-1, 1]$ on the y-axis, the graph to the left is generated.

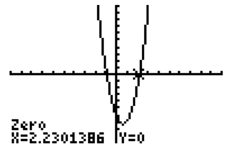


By using the $\boxed{\text{TRACE}}$ feature, we can conclude that one x-intercept is approximately 0.6 to the nearest tenth. By repeating these steps for the other x-intercept, we find that it is approximately -4.6.

Use a grapher to approximate the real number solutions to the nearest tenth. If an equation has no real number solution, state so.

1. $3x^2 - 4x - 6 = 0$ −0.9, 2.2
2. $x^2 - x - 9 = 0$ −2.5, 3.5
3. $2x^2 + x + 2 = 0$ no real solution
4. $-4x^2 - 5x - 4 = 0$ no real solution
5. $-x^2 + x + 5 = 0$ −1.8, 2.8
6. $10x^2 + 6x - 3 = 0$ −0.9, 0.3





$x = -0.897,$
2.23
zero
#2

$(-0.897, 0)$
$(2.23, 0)$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

−3, 5	$a = 0$ or $b = 0$	0	linear
3, −5	quadratic	1	

1. An equation that can be written in the form $ax^2 + bx + c = 0$, with $a \neq 0$, is called a ___quadratic___ equation.
2. If the product of two numbers is 0, then at least one of the numbers must be ___0___.
3. The solutions to $(x - 3)(x + 5) = 0$ are ___$x =$ 3, −5___.
4. If $a \cdot b = 0$, then ___$a = 0$ or $b = 0$___.

6.6 HW Assignment:

pg. 424: 1 - 95 (eoo)