

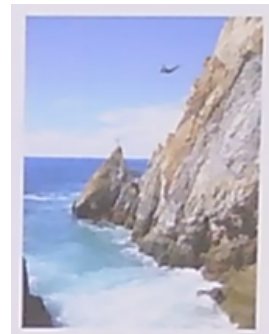
6.7 Quadratic Equations & Problem Solving

OBJECTIVE 1: Solving Problems Modeled by Quadratic Equations

Remember that these answers are real-world and therefore should **make sense**. For example, a person's age or the length of a rectangle is always a positive number. **Discard extraneous solutions** that do not make sense as a solution to the problem.

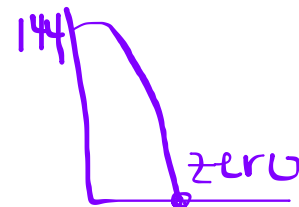
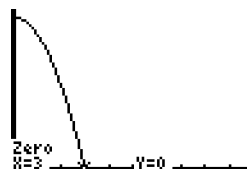
Example 1: Finding Free-Fall Time

Since the 1940s, one of the top tourist attractions in Acapulco, Mexico, is watching the La Quebrada cliff divers. The divers' platform is about 144 feet above the sea. These divers must time their descent just right, since they land in the crashing Pacific Ocean in an inlet that is at most 9.5 feet deep. Neglecting air resistance, the height h in feet of a cliff diver above the ocean after t seconds is given by the quadratic equation $h = -16t^2 + 144$. Find how long it takes the diver to reach the ocean.



```
WINDOW
Xmin=0
Xmax=10
Xscl=1
Ymin=0
Ymax=150
Yscl=1
↓Xres=1
```

3 secs



Practice 1:

Cliff divers also frequent the falls at Waimea Falls Park in Oahu, Hawaii. One of the popular diving spots is 64 feet high. Neglecting air resistance, the height of a diver above the pool after t seconds is $h = -16t^2 + 64$. Find how long it takes a diver to reach the pool.



Plot1	Plot2	Plot3	WINDOW
$\sqrt{Y1} = -16X^2 + 64$			Xmin=0
			Xmax=10
$\sqrt{V2} =$			Xscl=1
$\sqrt{V3} =$			Ymin=0
$\sqrt{V4} =$			Ymax=75
$\sqrt{V5} =$			Yscl=1
			Xres=1

2 SECS

(2, 0)

$$x = ? \quad y = 0$$

$$0 = -16(t^2 - 4)$$

$$0 = -16(t+2)(t-2)$$

$$t+2=0$$

$$t-2=0$$

$$t = -2$$

$$t = 2$$

Example 2: Finding an Unknown Number

The square of a number plus three times the number is 70. Find the number.

X	Y1
-12	5.3846
-11	6.5385
-10	7.7778
-9	9.1111
-8	10.5556
-7	12.1111

$$x^2 + 3x = 70$$

$$x^2 + 3x - 70 = 0$$

$$(x+10)(x-7) = 0$$

$$x+10=0$$

$$x-7=0$$

$$x = -10$$

$$x = 7$$

Practice 2:

The square of a number minus eight times the number is equal to forty-eight. Find the number.

$$x^2 - 8x = 48$$

$$x^2 - 8x - 48 = 0$$

$$(x-12)(x+4) = 0$$

$$x-12=0$$

$$x+4=0$$

$$x = 12$$

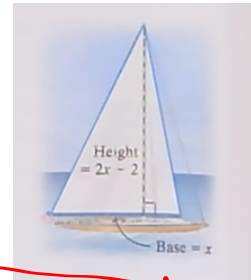
$$x = -4$$

X	Y1
-12	3.6923
-11	4
-10	4.3636
-9	4.9
-8	5.5556
-7	6.3571

Example 3: Find the Dimensions of a Sail

The height of a triangular sail is 2 meters less than twice the length of the base. If the sail has an area of 30 square meters, find the length of its base and the height.

30 m²



$$A = \frac{1}{2}bh$$

$$30 = \frac{1}{2}(x)(2x-2)$$

$$h = 2(b) - 2$$

$$= 12 - 2$$

$$= 10$$

$$30 = \frac{1}{2}(2x^2 - 2x)$$

$$30 = x^2 - x$$

$$0 = x^2 - x - 30$$

$$0 = (x-6)(x+5)$$

$$x-6=0 \quad x+5=0$$

$$\boxed{x=6} \quad x \neq -5$$

base: 6m
ht: 10m

Practice 3:

An engineering team from Georgia Tech earned second place in a flight competition, with their triangular shaped paper hang glider. The base of their prize-winning entry was 1 foot less than three times the height. If the area of the triangular glider wing was 210 square feet, find the dimensions of the wing.



$$h = x$$

$$b = 3x - 1$$

$$A = 210 \text{ ft}^2$$

$$\boxed{h = 12 \text{ ft}}$$

$$\boxed{b = 35 \text{ ft}}$$

$$A = \frac{1}{2}bh$$

$$2 \cdot 210 = \frac{1}{2}(3x-1)(x) \cdot 2$$

$$420 = 3x^2 - x$$

$$-420 \quad -420$$

$$0 = 3x^2 - x - 420$$

$$0 = (3x^2 - 36x) + (35x - 420)$$

$$= 3x(x-12) + 35(x-12)$$

$$= (3x+35)(x-12)$$

$$3x+35=0$$

$$3x = -35$$

$$x \neq -\frac{35}{3}$$

$$x-12=0$$

$$\boxed{x=12}$$

~~$$\begin{array}{r} a.c \\ -1260 \\ 35 \times -36 \\ -1 \end{array}$$~~

X	Y1
-35	36
-34	37.059
-33	38.118
-32	39.177
-31	40.236
-30	41.295
-29	42.354

X = -35

$$b = 3(12) - 1$$

$$= 36 - 1$$

$$= 35$$

Example 4: Finding Consecutive Even Integers

Find two consecutive even integers whose product is 34 more than their sum.

$$\begin{array}{l} 6, 8 \\ -6, -4 \end{array}$$

$$x(x+2) = 34 + x + x + 2$$

$$x^2 + 2x - 2x - 36 = 0$$

$$x^2 - 36 = 0$$

$$(x+6)(x-6) = 0 \quad x = -6, -6$$

Practice 4:

Find two consecutive integers whose product is 41 more than their sum.

$$x(x+1) = 41 + x + x + 1$$

$$x^2 + x - 42 - 2x = 0$$

$$x^2 - x - 42 = 0$$

$$(x-7)(x+6) = 0$$

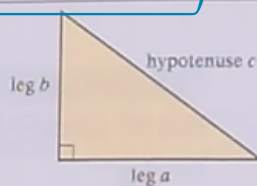
$$x = 7, -6$$

$$\begin{array}{l} 7, 8 \\ -6, -5 \end{array}$$

Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse.

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2 \quad \text{or} \quad a^2 + b^2 = c^2$$

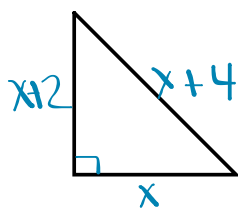
**Helpful Hint**

If you use this formula, don't forget that c represents the length of the hypotenuse.

$$l_1^2 + l_2^2 = h^2$$

Example 5: Finding the Dimensions of a Triangle

Find the lengths of the sides of a right triangle if the lengths can be expressed as three consecutive even integers.



$$(x)^2 + (x+2)^2 = (x+4)^2$$

$$x^2 + x^2 + 4x + 4 = x^2 + 8x + 16$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

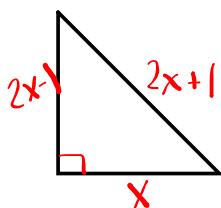
$$x = 6 \quad x \neq -2$$

$$\boxed{6, 8, 10}$$

$$\Rightarrow 15x^2$$

Practice 5:

Find the dimensions of a right triangle where the second leg is 1 unit less than double the first leg, and the hypotenuse is 1 unit more than double the length of the first leg.



$$\boxed{8, 15, 17}$$

$$(x)^2 + (2x-1)^2 = (2x+1)^2$$

$$x^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$

$$x \neq 0 \quad \boxed{x=8}$$

6.7 HW Assignment

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