6.7 Recursively Defined Sequences

Essential Question:

How can you define a sequence recursively?

Core Vocabulary:

explicit rule

recursive rules

arithmetic sequence

geometric sequence

What You Will Learn:

- Write terms of recursive defined sequences.
- Write recursive rules for sequences.
- Translate between recursive rules and explicit rules.
- Write recursive rules for special sequences.

Feb 8-6:07 PM

Explicit rule: (algebraic) ... tells how to find a term based on the position of that term

arithmetic ... d

$a_n = a_1 + d(n-1)$

geometric ... r

$a_n = a_1 \cdot r^{n-1}$

Recursive rule: tells how to find a term based on the term prior to this term

$50^{th} \rightarrow$ use 49^{th}

$17^{th} \rightarrow$ use 16^{th}

If you're on the 5th term, what is the previous term?

If you're on the 22nd term, what is the previous term?

22-1 → 21st

If you're on the 8th term, what is the previous term?

8-1 → 7th

What term is previous to the nth term? → (n-1)st

$a_n = 5 + a_{n-1}$

any nth term

previous term to the nth term

Feb 8-6:12 PM

Example: 4, 7, 10, 13, 16 ...

a_1 +3 +3 +3

explicit rule (algebraic):
arithmetic
b< d = +3

$a_n = a_1 + d(n-1)$

$a_n = 4 + 3(n-1)$

first recursive rule:
finding the pattern

$a_n = a_{n+1} + 3$

$a_n = a_{n+1}$ — do what?

new previous

Feb 8-6:11 PM

Example: 4, 20, 100, 500 ...

×5 ×5 ×5

geometric $r = 5$

explicit rule (algebraic): $a_n = a_1 \cdot r^{n-1}$

$a_n = 4 \cdot 5^{n-1}$

$a_1 = 4$

recursive rule: $a_n = a_{n-1} \cdot 5$

Feb 8-6:11 PM

Graphing Example

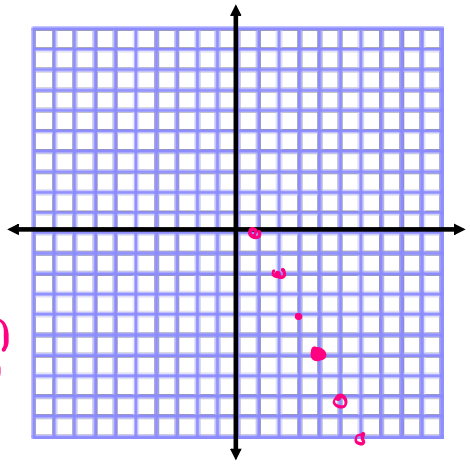
1. Write the first 6 terms of the sequence and graph it.

$a_1 = 0$, $a_n = a_{n-1} - 2$ ← recursive

previous

n	$a_{n-1} - 2$	a_n
1		0
2	0 − 2	−2
3	−2 − 2	−4
4	−4 − 2	−6
5	−6 − 2	−8
6	−8 − 2	−10

(1, 0)
(2, −2)
(3, −4)
(4, −6)
(5, −8)
(6, −10)



arithmetic $d = -2$

→ linear slope: −2

Feb 8-6:12 PM

2. Write a recursive rule for the sequences:

↳ find the pattern

a. 8, 3, -2, -7, -12 ...

-5 -5 -5 -5

anyterm = prev. term - 5

$a_n = a_{n-1} - 5$

subscripts

~~a_{n-1}~~ a_n ~~a_{n-1}~~ a_{n-1}

b. 128, -32, 8, -2, 0.5, ...

÷-4 ÷-4

$r = \frac{1}{4}$

geometric

$a_n = a_{n-1} \cdot \left(-\frac{1}{4}\right)$

Feb 8-6:20 PM

3. Write an explicit rule for the recursive rules:

a. $a_1 = -45$, $a_n = a_{n-1} + 20$

NT PT

-45, -25, -5, 15 ...

+20 +20 +20

arithmetic

$d = 20$

$a_n = a_1 + d(n-1)$

$a_n = -45 + 20(n-1)$

simplified

$= -45 + 20n - 20$

$a_n = -65 + 20n$

b. $a_1 = 13$, $a_n = -3a_{n-1}$

new PT

13, -39, 117, -351 ...

·-3 ·-3 ·-3

geometric

$r = -3$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 13(-3)^{n-1}$

Feb 8-6:25 PM

4. Write a recursive rule for the explicit rules:
(find) (given)

a. $a_n = -n + 1$

1st	2nd	3rd
n=1	n=2	n=3
-1+1=0	-2+1	-3+1
0	-1	-2
(1,0)	(2,-1)	(3,-2)

0, -1, -2, -3, . . .
-1 -1 -1
arithmetic : d = -1
$a_n = a_{n-1} - 1$

b. $a_n = -2.5(4)^{n-1}$

1st	2nd	3rd
n=1	n=2	n=3
$-2.5(4)^{1-1}$	$-2.5(4)^{2-1}$	$-2.5(4)^{3-1}$
-2.5	-10	-40
(1, -2.5)	(2, -10)	(3, -40)

-2.5, -10, -40, . . .
·4 ·4
geometric : r = -4
$a_n = (a_{n-1}) \cdot 4$

Feb 8-6:31 PM

6.7 Assignment:

6.7 Extra Practice WS

Feb 8-6:34 PM