

OBJECTIVE 1: Rational Function

Is a function in the form $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.

STEPS:

1. Determine if the function is bottom heavy, top heavy, or equal; heavy refers to the Degree or highest exponent
 - a. Bottom heavy (KK): means $y = 0$ is the HA
 - b. Top heavy (DP): means synthetic or long division is used to find your slant HA
 - c. Equal: means you use the leading coefficients as your HA
2. Factor anything that is factorable
3. Canceling any factors results in a hole
 - a. Holes are coordinates
 - b. The x value comes from the factor that cancels
 - c. The y value comes from plugging the x-value back into the remaining function
4. Vertical asymptote: $x = -\frac{d}{c}$
5. Horizontal asymptote: $y = \frac{a}{c}$
6. Domain restriction: $cx + d = 0$

$$y = \frac{ax + b}{cx + d}$$

$$y = \frac{x}{x-2}$$

TASK 1: Determine if the provided function is bottom, top, or equal heaviness to determine the horizontal asymptotes. If there are holes, state your hole.

a) $f(x) = \frac{1}{x}$ \circ \mid

$$y=0$$

b) $f(x) = \frac{2}{x+1}$ \circ

$$y=0$$

c) $f(x) = \frac{1}{x^2}$ \circ $\underline{2}$

$$y=0$$

d) $f(x) = \frac{2}{x-3}$ \circ \mid

$$y=0$$

$$e) f(x) = \frac{x}{x+1}$$

$$y=1$$

$$f) f(x) = \frac{2x}{x-1}$$

$$y=2 = z$$

$$g) f(x) = \frac{x}{2x+1}$$

$$y=\frac{1}{2}$$

$$h) f(x) = \frac{6x}{3x-1}$$

$$y=\frac{6}{3} = 2$$

$$i) f(x) = \frac{x^2+x}{x+1}$$

$$= \frac{-x(x+1)}{x+1}$$

$$= -x \quad (-1, -1)$$

$$\begin{aligned} x+1 &= 0 \\ x &= -1 \\ (-1)^2 - 1 &= 0 \end{aligned}$$

$$j) f(x) = \frac{x^2+3x}{x}$$

$$= \frac{x(x+3)}{x}$$

$$= x+3 \quad (0, 3)$$

$$x=0 \quad 0+3=3$$

$$k) f(x) = \frac{x^2-x-2}{x-2}$$

$$= \frac{(x-2)(x+1)}{(x-2)}$$

$$= x+1 \quad (2, 3)$$

$$2+1=3$$

$$x-z=0$$

$$x=z$$

$$l) f(x) = \frac{x^2+x}{x^2-2x}$$

$$= \frac{x(x+1)}{x(x-2)}$$

$$= \frac{x+1}{x-2} \quad (0, -\frac{1}{2})$$

$$x=0 \quad 0+1 = -\frac{1}{0-2} = \frac{1}{2}$$

$$m) f(x) = \frac{x^2}{x+1}$$

$$\begin{array}{r} -1 \mid 1 \ 0 \ 0 \\ + \quad -1 \ 1 \\ \hline 1 \ -1 \ 1 \end{array}$$

$$y=x-1$$

$$n) f(x) = \frac{2x^2}{x-1}$$

$$\begin{array}{r} 1 \mid 2 \ 0 \ 0 \\ + \quad -2 \ 2 \\ \hline 2 \ 2 \ 2 \end{array}$$

$$y=2x+2$$

$$o) f(x) = \frac{x^2+4}{x+2}$$

$$\begin{array}{r} -2 \mid 1 \ 0 \ 4 \\ + \quad -2 \ 4 \\ \hline 1 \ -2 \ 8 \end{array}$$

$$y=x-2$$

Common mistakes:

equal: coefficients

Hole: cancels; factors, (x,y)

Bottom: y=0

Top: slant asymptote

Still need help with:

long/synthetic division