7.2 Standard Units and Areas Under the Standard Normal Distribution

Essential Question:

- How do we compare data and test hypotheses of different units?

Focus Points:

- Given μ and σ, convert raw data to z scores.
- Given μ and σ, convert z scores to raw data.
- Graph the standard normal distribution, and find areas under the standard normal curve.

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The **z value** or **z score** (also know as the standard score) gives the number of standard deviations between the original measurement x and the mean μ and the x distribution.

$$z = \frac{x - \mu}{\sigma}$$

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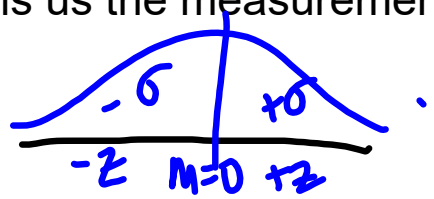
WHAT DOES A STANDARD SCORE TELL US?

A ***standard score*** ***OR*** ***z score*** of a measurement tells us the number of standard deviations the measurement is from the mean.

- A standard score close to zero tells us the measurement is near the mean of the distribution.
- A positive standard score tells us the measurement is above the mean.
- A negative standard score tells us the measurement is below the mean.



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Example 1: Happy Joe's Pizza

Happy Joe's Pizza place specifies that the average (mean) amount of cheese on a large pizza should be 8 oz and the standard deviation only 0.5 oz. An inspector picks out a large pizza at random in one of the pizza locations and finds that it is made with 6.9 ounces of cheese. Assume that the amount of cheese on a pizza follows a normal distribution. If the amount of cheese is below the mean by more than 3 standard deviations, the location will be in danger of losing its Happy Joe's franchise.

How many standard deviations from the mean is 6.9? Is the location in danger of losing its franchise?

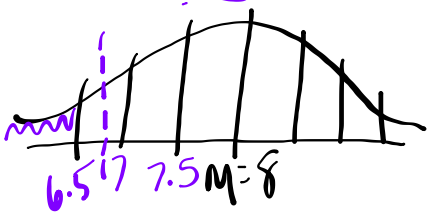
$$z = \frac{x - \mu}{\sigma}$$

x = 6.9

μ = 8

σ = 0.5

$$z = \frac{6.9 - 8}{.5} = \frac{-1.1}{.5} = -2.2$$



Pull

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Working Backwards

Given an x distribution with mean μ and standard deviation σ, the raw scores x corresponding to a z score is

$z = \frac{x - \mu}{\sigma}$

$z\sigma = x - \mu$

$z\sigma + \mu = x$

$$x = z\sigma + \mu$$

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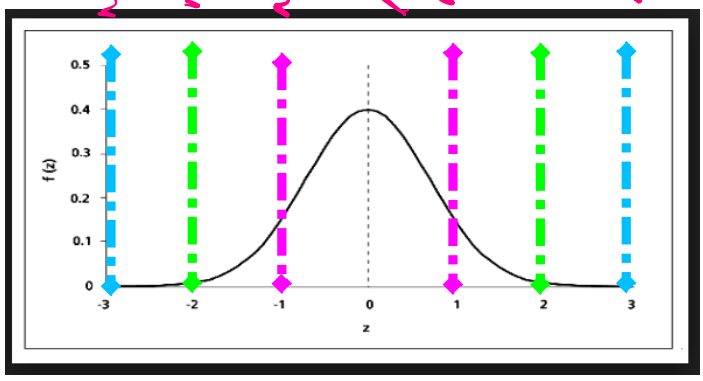
STANDARD NORMAL DISTRIBUTION

The standard normal distribution is a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.



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WHAT DOES THE STANDARD NORMAL DISTRIBUTION TELL US?

When we have the standard normal distribution, we know

- the mean is 0.
- the standard deviation is 1.
- any normal distribution can be converted to a standard normal distribution by converting all the measurements to standard z scores.

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USING THE Z-SCORE TABLE

Table 3 in the Appendix

a) Find the area under the standard normal curve to the left of z = -1.

0.1587 Pull

-1.00

0.1587

b) Find the area to the left of z = 1.18.

0.8810 Pull

.1190

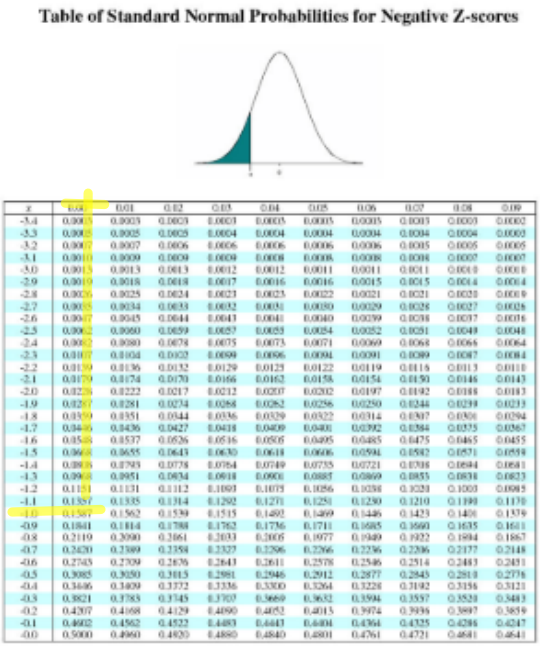
Table of Standard Normal Probabilities for Negative Z-scores

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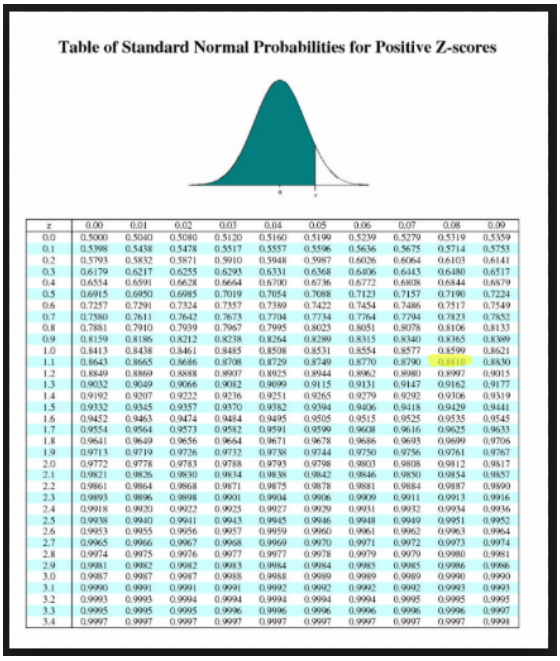
Table of Standard Normal Probabilities for Positive Z-scores

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

c) Find z = -1.00 using this table.

d) Find z = 1.18 using this table.

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HOW TO USE A LEFT-TAIL STYLE STANDARD NORMAL DISTRIBUTION TABLE

1. For areas to the left of a specified z value, use the table entry directly.

2. For areas to the right of a specified z value, look up the table entry for z and subtract the area from 1. OR use the symmetry of the normal curve and look up the table entry for -z.

3. For areas between two z values, z_1 and z_2 (where $z_2 > z_1$), subtract the table area for z_1 from the table area for z_2.

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For using Table 3, treat any area to the left of -3.49 as 0 and to the right of 3.49 as 0!!

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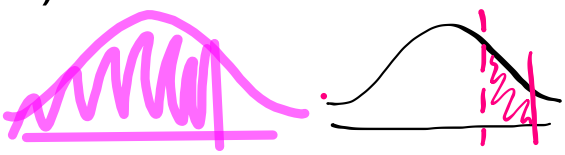
Example 2: Using the z-score table

a) Find the area between z = 1 and z = 2.70



$P(z=2.70) - P(z=1)$

$P(0.9965) - P(0.8413)$

0.1552

Standard Normal Probabilities

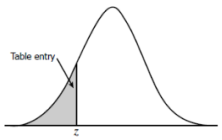


Table entry for z is the area under the stan
to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150

TABLE II (cont.)
Areas under the standard normal curve

z	Second decimal place in z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
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0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
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1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

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b) Find the area to the right of z = 0.94.

TABLE II (cont.)
Areas under the standard normal curve

z	Second decimal place in z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
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1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
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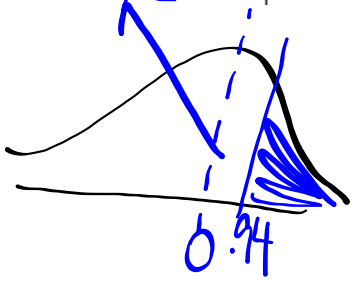
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

0.8264



$1 - 0.8264$

$= 0.1736$

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HW: pg. 309: 1, 3, 5, 9, 11 - 49 (o)

1. The # of standard deviations from the mean.

3. 0

5. a) -1 b) 2.4 c) 20 d) 36.5

9. a) Robert, Juan, Linda b) Joel c) Susan & Jan

d) Robert, 172; Juan, 184; Susan, 110; Joel, 150; Jan, 134; Linda 182

11. a) $z > -1$ b) $z < -2$ c) $-2.67 < z < 2.33$ d) $x < 4.4$ e) $x > 5.2$

f) $4.1 < x < 4.5$ g) Yes, $z = 3.67$

13. 0.5000 15. 0.0934 17. 0.6736 19. 0.0643 21. 0.888 23. 0.4993

25. 0.8953 27. 0.3471 29. 0.0306 31. 0.5000 33. 0.4483 35. 0.8849

37. 0.0885 39. 0.8849 41. 0.8808 43. 0.3226 45. 0.4474 47. 0.2939

49. 0.6704

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