7.3 Special Products (Day 1)

Essential Question What are the patterns in the special products $(a+b)(a-b)$, $(a+b)^2$, and $(a-b)^2$?

The 3 products above show up so many times in math that we just want to memorize what they equal. Let's see if we can figure them out by looking at some patterns, starting with the first product.

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(a + b)(a - b) is called a sum times a difference, why?

The following 3 problems shows the sum of 2 numbers multiplied by their difference.

FOIL out all 3 and see if you notice any repeated reasoning.

1) (x + 3)(x - 3)

$x^2 \cancel{-3x} \cancel{+3x} - 9$

$\boxed{x^2 - 9}$

2) (m - 10)(m + 10)

$m^2 \cancel{+10m} \cancel{-10m} - 100$

$\boxed{m^2 - 100}$

3) (j + 7)(j - 7)

$j^2 \cancel{-7j} \cancel{+7j} - 49$

$\boxed{j^2 - 49}$

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Let's see if this works when we know both numbers.

1) Use FOIL to simplify: (8 - 2)(8 + 2)

$64 + \cancel{16} - \cancel{16} - 4 = \boxed{60}$

2) Use the pattern to simplify: (8 - 2)(8 + 2)

(What was the pattern?)

$64 - 4 = \boxed{60}$

3) Use order of ops to simplify: (8 - 2)(8 + 2)

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$(6)(10) = \boxed{60}$

The reason we usually can't use order of ops is because you will not know both numbers.

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So in general a sum times a difference of the same 2 numbers is equal to the difference of their squares:

$$\boxed{(a + b)(a - b) = a^2 - b^2}$$

Use the pattern to answer:

1) (w + 6)(w - 6)

$w^2 - 36$

2) (x - 12)(x + 12)

$x^2 - 144$

3) (4 - x)(4 + x)

$16 - x^2$

4) $\left(\frac{1}{3}m + \frac{2}{5}\right)\left(\frac{1}{3}m - \frac{2}{5}\right)$

$\frac{1}{9}m^2 - \frac{4}{25}$

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The next special product is the **square of a sum (or square of a binomial):** $(a + b)^2$. This means 2 factors of the base $(a+b)^2 = (a+b)(a+b)$.

$xy = yx$

Let's discover the pattern using FOIL.

1) $(x + 8)^2$

$(x+8)(x+8)$

$x^2 + 8x + 8x + 64$

$\boxed{x^2 + 16x + 64}$

2) $(v + 3)^2$

$(v+3)(v+3)$

$v^2 + 3v + 3v + 9$

$\boxed{v^2 + 6v + 9}$

3) $(x + y)^2$

$(x+y)(x+y)$

$x^2 + xy + xy + y^2$

$\boxed{x^2 + 2xy + y^2}$

It looks like the product ends up with three terms (called a trinomial) where the first term is the 1st thing squared, the middle term is two times the 1st & 2nd thing and the last term is the last thing squared.

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Let's verify our pattern using all numbers:

1) by FOIL

$(5 + 3)^2$

$(5+3)(5+3)$

$25 + 15 + 15 + 9$

$25 + 30 + 9$

$\boxed{64}$

2) by pattern

$(a+b)^2$

$(5 + 3)^2$

$a^2 + 2ab + b^2$

$5^2 + 2(5)(3) + (3)^2$

$25 + 30 + 9$

$\boxed{64}$

3) order of ops

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$(5 + 3)^2$

$(8)^2$

$\boxed{64}$

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So, in general: $\boxed{(a + b)^2 = a^2 + 2ab + b^2}$.

From now on we want to use this special pattern to square any binomial.

1) $(x + 11)^2$

$a = x \quad b = 11$

$x^2 + 2(x)(11) + 11^2$

$\boxed{x^2 + 22x + 121}$

2) $(g + 5)^2$

$a = g \quad b = 5$

$g^2 + 2(g)(5) + 5^2$

$\boxed{g^2 + 10g + 25}$

3) $(3x + 6y)^2$

$a = 3x \quad b = 6y$

$(3x)^2 + 2(3x)(6y) + (6y)^2$

$\boxed{9x^2 + 36xy + 36y^2}$

4) $\left(\frac{1}{2}b + \frac{3}{4}\right)^2$

$a = \frac{1}{2}b \quad b = \frac{3}{4}$

$\left(\frac{1}{2}b\right)^2 + \frac{2}{1}\left(\frac{1}{2}b\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2$

$\frac{1}{4}b^2 + \frac{6}{8}b + \frac{9}{16}$

$\boxed{\frac{1}{4}b^2 + \frac{3}{4}b + \frac{9}{16}}$

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The next special product is the square of a difference (or square a binomial): $(a - b)^2$.

This means 2 factors of the base $(a-b)^2 = (a-b)(a-b)$.

Let's discover the pattern using FOIL.

1) $(x - 8)^2$

$(x-8)(x-8)$

$x^2 - 8x - 8x + 64$

$\boxed{x^2 - 16x + 64}$

2) $(v - 3)^2$

$(v-3)(v-3)$

$v^2 - 3v - 3v + 9$

$\boxed{v^2 - 6v + 9}$

3) $(x - y)^2$

$(x-y)(x-y)$

$x^2 - xy - xy + y^2$

$\boxed{x^2 - 2xy + y^2}$

It looks like the product ends up with 3 terms (called a trinomial) where the first term is the 1st thing squared, the middle term is −2 times 1st & 2nd thing and the last term is the last thing squared. (Is this the same pattern as the square of a sum?)

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Let's verify our pattern using all numbers: REMDAS

1) by FOIL

$(5 - 3)^2$

$(5-3)(5-3)$

$25-15-15+9$

$25-30+9$

$\boxed{4}$

2) by pattern

$(5 - 3)^2$

$a^2-2ab+b^2$

$5^2-2(5)(3)+(-3)^2$

$25-30+9$

$\boxed{4}$

3) order of ops

$(5 - 3)^2$

$(2)^2$

$\boxed{4}$

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So, in general: $(a - b)^2 =$ $a^2-2ab+b^2$.

From now on we want to use this special pattern to square any binomial.

1) $(x - 9)^2$

$x^2-2(x)(9)+9^2$

$\boxed{x^2-18x+81}$

2) $(g - 15)^2$

$g^2-2(g)(15)+15^2$

$\boxed{g^2-30g+225}$

3) $(2x - 5y)^2$

$(2x)^2-2(2x)(5y)+(5y)^2$

$\boxed{4x^2-20xy+25y^2}$

4) $\left(\frac{1}{3}b - \frac{1}{2}\right)^2$

$\left(\frac{1}{3}b\right)^2-2\left(\frac{1}{3}b\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2$

$\boxed{\frac{1}{9}b^2-\frac{1}{3}b+\frac{1}{4}}$

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Fill in the 3 Special Products use a and b:

1) Sum x Difference:

$$(a+b)(a-b)=(a^2-b^2)$$

2) Square of a Sum:
(square of a binomial)

$$(a+b)^2=a^2+2ab+b^2$$

3) Square of a Difference:
(square of a binomial)

$$(a-b)^2=a^2-2ab+b^2$$

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7.3 Day 1 Special Products

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