7.3 Areas Under Any Normal Curve

Essential Question:

- How do I determine the probability of something occurring given data with different units?

Focus Points:

- Compute the probability of "standardized events."
- Find a z-score from a given normal probability (inverse normal).
- Use the inverse normal to solve guarantee problems.

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HOW TO WORK WITH NORMAL DISTRIBUTIONS

To find areas and probabilities for a random variable *x* that follows a normal distribution with mean μ and standard deviation σ, convert *x* values to *z* values using the formula

$$z = \frac{x - \mu}{\sigma}$$

Then use Table 3 of the Appendix page A8 to find corresponding areas and probabilities.

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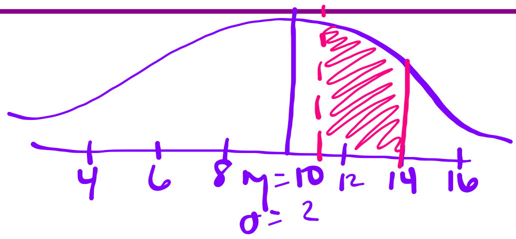
Example 1:

Let *x* have a normal distribution with μ = 10 and σ = 2. Find the probability that an *x* value selected at random from this distribution is between 11 and 14. In symbols, find P(11 ≤ x ≤ 14).

1. Graph your curve to get a visual
2. Find two z-scores
3. Subtract the smaller from the bigger value



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2. $\frac{14-10}{2} \leq z \leq \frac{11-10}{2}$

$2 \leq z \leq 0.5$

3. .9772 − .6915 = .2857

$P(11 \leq x \leq 14) = 28.57\%$

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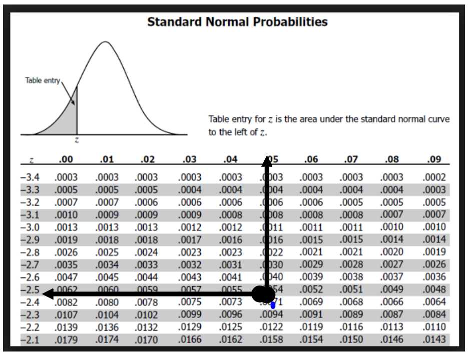
Guarantee Period = Inverse Normal

Finding x or z given a probability = working backwards

REALWORLD: A company needs to guarantee the products will last as long as the duration of the guarantee period.

work from inside the table out



Standard Normal Probabilities

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	[illegible]	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	[illegible]	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	[illegible]	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	[illegible]	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	[illegible]	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	[illegible]	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	[illegible]	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	[illegible]	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	[illegible]	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	[illegible]	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	[illegible]	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143

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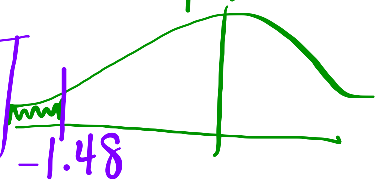
Example 2: Video Games

Magic Video Games, Inc., sells an expensive video game package. Because the package is so expensive, the company wants to advertise an impressive guarantee for the life expectancy of its computer control system. The guarantee policy will refund the full purchase price if the computer fails during the guarantee period. The research department has done tests that show that the mean life for the computer is 30 months, with standard deviation of 4 months. The computer life is normally distributed. How long can the guarantee period be if management does not want to refund the purchase price on more than 7% of the Magic Video packages?

What percentage are we working backwards with? .07

Find it inside the z-score table on page A8.

$-1.4 \longrightarrow .0708 \quad .0694$ (0.08)



$z = -1.48$

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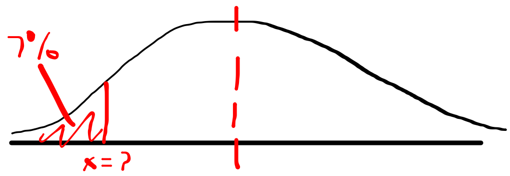

$z = -1.48$, $\mu = 30$, $\sigma = 4$

$z = \frac{x - \mu}{\sigma}$

$-1.48 = \frac{x - 30}{4}$

$-5.92 = x - 30$

$+30 \quad +30$

$24.08 = x$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

$x = z\sigma + \mu$

$= -1.48\,(4) + 30$

$= 24.08$ months

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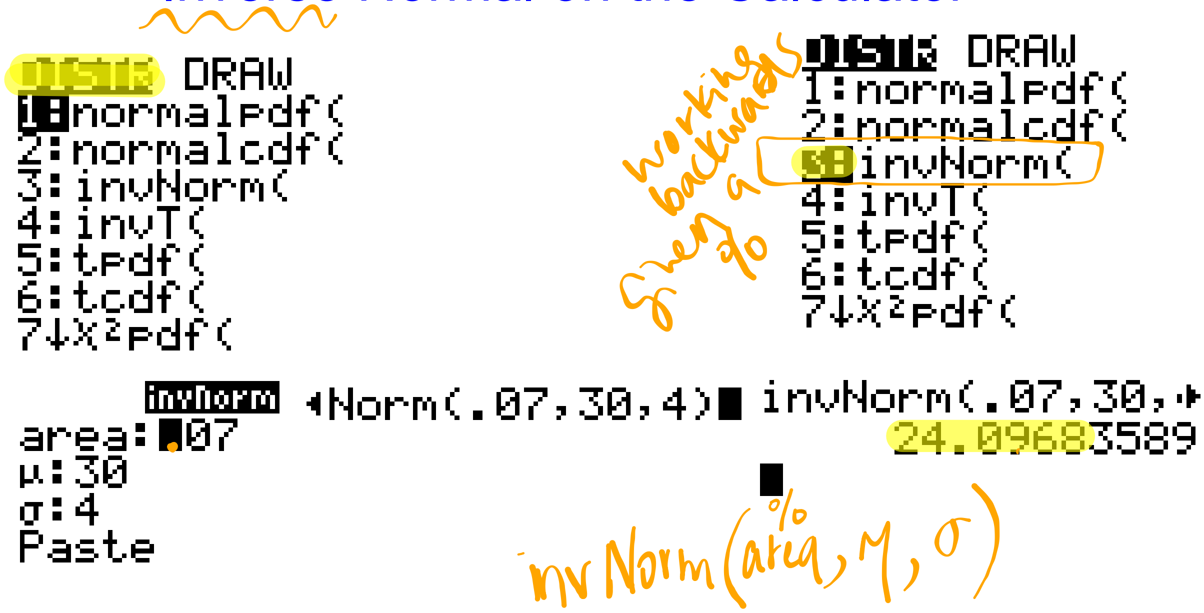
Inverse Normal on the Calculator



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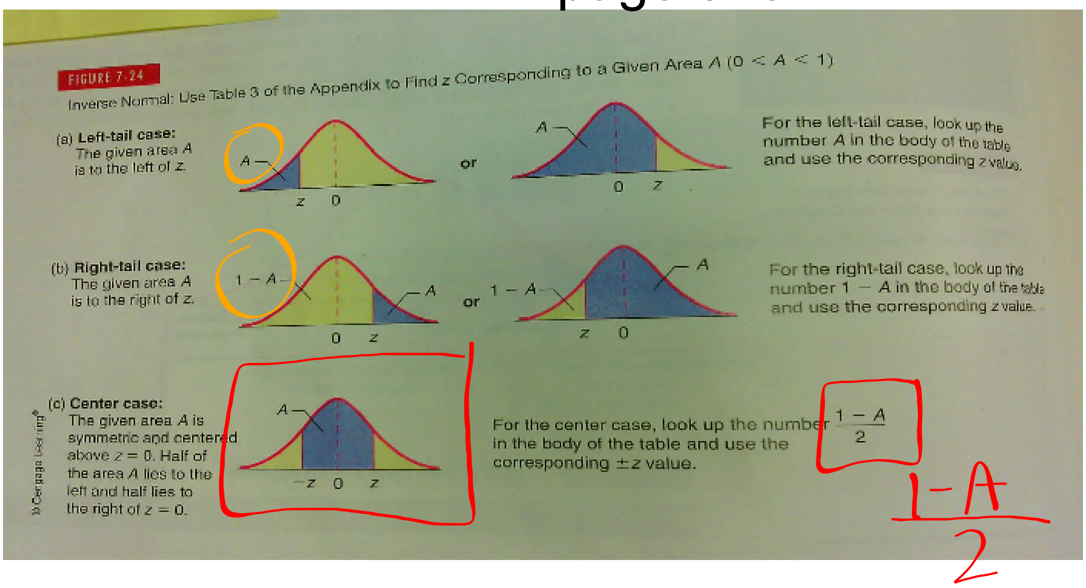

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Example 3:

Find the z value such that 90% of the area under the standard normal curve lies between -z and z.

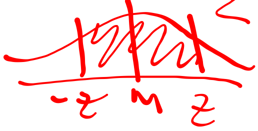
(scroll down for answers!)

1. draw a sketch.
2. pick the formula to use to find the probability.
3. locate that probability inside the table on A8
4. determine the corresponding z-score(s)

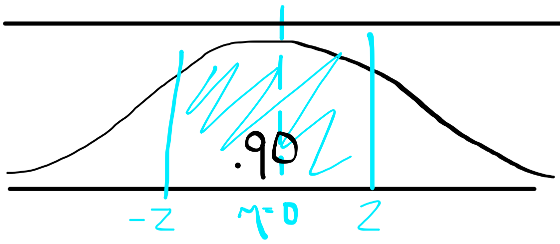
$A = .9$

between

$\pm z = \frac{1-A}{2}$



$\pm z = \frac{1-.9}{2} = \frac{.1}{2} = .05$



$$\frac{1-.9000}{2} = 0.0500$$

between 0.0495 and 0.0505 so

z = -1.65 and -1.64 so...z = -1.645

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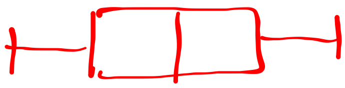
HOW TO DETERMINE WHETHER DATA HAVE A NORMAL DISTRIBUTION

1. **Histogram**: Make a histogram. For a normal distribution, the histogram should be roughly bell-shaped.

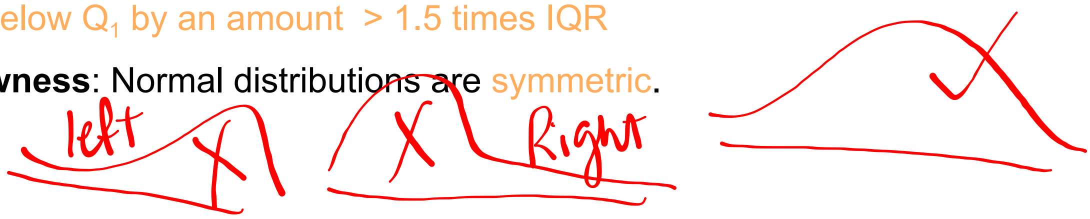
2. **Outliers**: For a normal distribution, there should not be more than ONE outlier. One way to check for outliers is to use a box-and-whisker plot. or create a MODIFIED box plot. Recall that outliers are those data values that are



above Q_3 by an amount > 1.5 times IQR

below Q_1 by an amount > 1.5 times IQR

3. **Skewness**: Normal distributions are symmetric.



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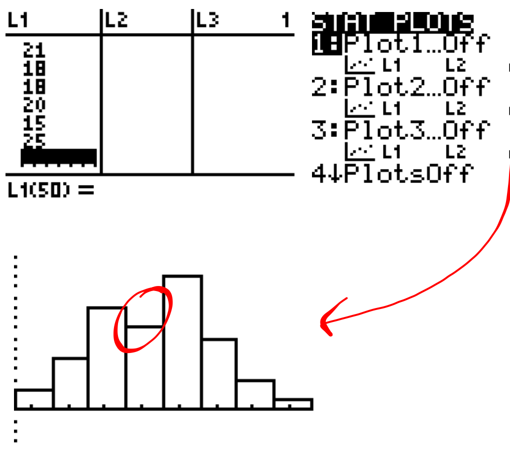
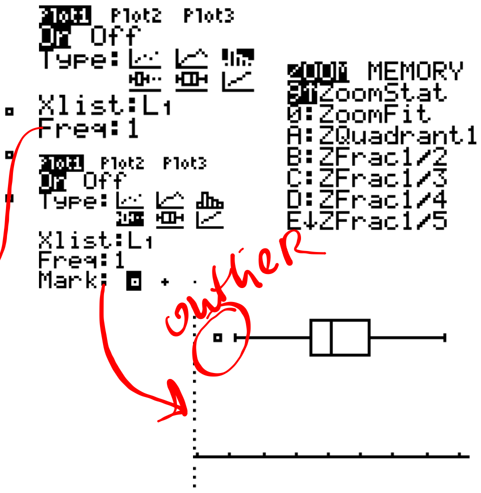
Example 4:

Consider the following data, which are rounded to the nearest integer.

19	19	19	16	21	14	23	17	19	20	18	24
20	13	16	17	19	18	19	17	21	24	18	23
19	21	22	20	20	24	17	20	22	19	22	21
18	20	22	16	15	21	23	21	18	18	20	15 25

Assess if the data provided represent a normal distribution. Use at least two methods to prove normalcy.





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HW: pg. 321: 1 - 29 (o)

1. 0.50
3. Negative.
5. 0.5328
7.0.2286
9. 0.1593
11. 0.0016
13. 0.7486
15. -1.555
17. 0.13
19. 1.41
21. -0.92
23. ± 2.33
25. a) 0.8413 b) 0.8413 c) 0.6826 d) 0.0139
27. a) 0.0099 b) 0.0174 c) 0.9727
29. a) 0.1292; replace 13% b) 34.76; guarantee for 35 months

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