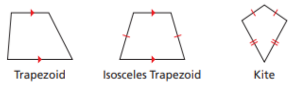
7.5 Properties of Trapezoids and Kites with work

7.5 Properties of Trapezoids and Kites



Name the properties of each of the following polygon.

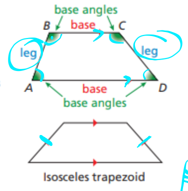
1) parallelogram (5)
2) rhombus (3)
3) rectangle (2)
4) square (10)

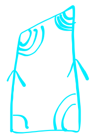
Using Properties of Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

Base angles of a trapezoid are two consecutive angles whose common side is a base. A trapezoid has two pairs of base angles. For example, in trapezoid *ABCD*, ∠*A* and ∠*D* are one pair of base angles, and ∠*B* and ∠*C* are the second pair. The nonparallel sides are the **legs** of the trapezoid.

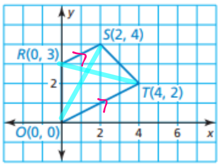
If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.





Example:

Show that *ORST* is a trapezoid. Then decide whether it is isosceles.



$m\overline{RO}$ = undefined

$m\overline{RS} = \frac{1}{2}$

$m\overline{ST} = -1$

$m\overline{TO} = \frac{2}{4} = \frac{1}{2}$

$RO = ST$

$RO = \sqrt{(0-0)^2+(3-0)^2}$

$= \sqrt{9} = 3u$

$ST = \sqrt{(2-4)^2+(4-2)^2}$

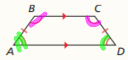
$= \sqrt{(-2)^2+(2)^2}$

$4+4 = \sqrt{8} \neq 3$

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

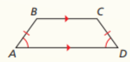
If trapezoid *ABCD* is isosceles, then ∠*A* ≅ ∠*D* and ∠*B* ≅ ∠*C*.



Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

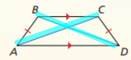
If ∠*A* ≅ ∠*D* (or if ∠*B* ≅ ∠*C*), then trapezoid *ABCD* is isosceles.



Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

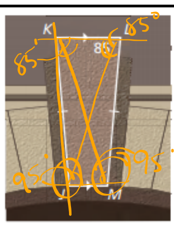
A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



Practice of Properties:

The stone above the arch in the diagram is an isosceles trapezoid. Find *m*∠*K*, *m*∠*M*, and *m*∠*J*.



$m\angle K = 85°$

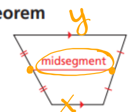
$m\angle M = 95°$

$m\angle J = 95°$

Using the Trapezoid Midsegment Theorem

READING
The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs. The theorem below is similar to the Triangle Midsegment Theorem (Thm. 6.8).

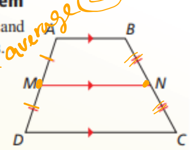


Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

average (2)

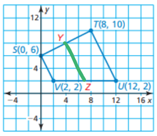
If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.



7.5 Properties of Trapezoids and Kites with work

Midsegment practice

Find the length of midsegment $\overline{YZ}$ in trapezoid *STUV*.



$SV = \sqrt{(2-0)^2+(2-6)^2}$

$= \sqrt{4+16} = \sqrt{20}$

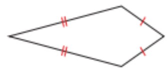
$TU = \sqrt{(12-8)^2+(2-10)^2}$

$= \sqrt{16+64} = \sqrt{80}$

$\boxed{YZ \approx 6.708u}$

Kites:

A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

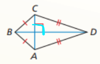


Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.

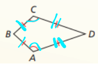
Proof p. 401



Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

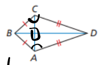
If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.



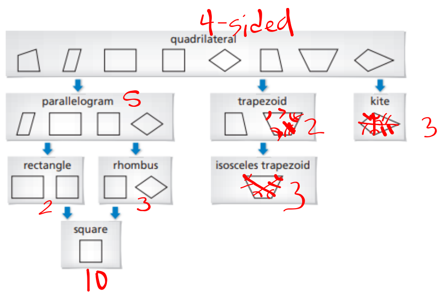
PROOF Kite Diagonals Theorem

Given *ABCD* is a kite, $\overline{BC} \cong \overline{BA}$, and $\overline{DC} \cong \overline{DA}$.

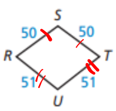
Prove $\overline{AC} \perp \overline{BD}$



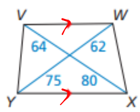
Statements	Reasons
1. ABCD is a kite; $\overline{BC} \cong \overline{BA}$, $\overline{DC} \cong \overline{DA}$	1. given
2. $\overline{AC} \perp \overline{BD}$	2. Kite Diagonal Thm



Give the most specific name for the quadrilateral. Explain your reasoning.



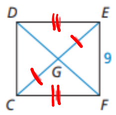
kite



$64+80 \neq 75+62$

$144 \neq 137$

trapezoid



Rect.

7.5 HW: pg. 403: 3, 7, 9, 11, 13, 15, 19, 29