8.3 Graph $f(x) = ax^2 + bx + c$

<u>Essential Question:</u>

How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

For Your Information

All of the quadratic functions that students have graphed have been symmetric about the y-axis. In the equation $y = ax^2 + c$, the values of a and c do not affect the graph in a horizontal direction. In this section, quadratic functions contain an x-term, which shifts the axis of symmetry away from the y-axis.

Feb 23-4:52 PM

What You Will Learn:
- Graph quadratic functions of the form $f(x) = ax^2 + bx + c$
- Find maximum and minimum values of quadratic functions.

Core Vocabulary:

maximum value   y

minimum value   y

Previous:

independent variable   x

dependent variable   y

8.3 Graph ax2+bx+c with work

TI- 84 STEPS to find the Max/Min

1) Press Calc (2nd Trace)

2) Choose #3 Minimum or #4 Maximum

3) For the Lower Bound, arrow to the left of the extremum and press enter.

4) For the Upper Bound, arrow to the right of the extremum and press enter.

5) For the Guess, arrow to the extremum and press enter.

6) The TI-84 will return a value for x and y. The x is where the extremum occurs and the y is the minimum or maximum value.

For other TI's: https://people.richland.edu/james/ti82/minmax/

Note that the calculator generated a vertex of (2.4999994, 105). Calculated by hand, the vertex is (2.5, 105). You need to know how to interpret results such as these and know that the vertex is indeed (2.5, 105).
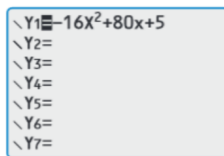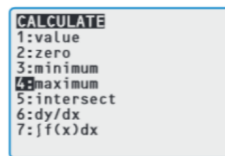
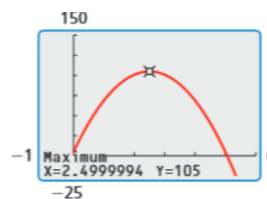|		
\Y1■−16X²+80x+5 \Y2= \Y3= \Y4= \Y5= \Y6= \Y7=	**CALCULATE** 1:value 2:zero 3:minimum 4:maximum 5:intersect 6:dy/dx 7:∫f(x)dx	150 −1 Maximum 6 X=2.4999994 Y=105 −25
Figure 1	**Figure 2**	**Figure 3**

Feb 23-5:09 PM

MEMORIZE

Core Concept

Graphing $f(x) = ax^2 + bx + c$

• The graph opens up when $a > 0$, and the graph opens down when $a < 0$.

• The y-intercept is c.

• The x-coordinate of the vertex is $-\dfrac{b}{2a}$.

• The axis of symmetry is $x = -\dfrac{b}{2a}$.

→ Standard form

$x = -\dfrac{b}{2a}$

$f(x) = ax^2 + bx + c$, where $a > 0$

$(0, c)$

vertex

x-intercepts
roots
zeros
Solutions

Axis of Symmetry

$-\dfrac{b}{2a}$

Feb 23-5:11 PM

2

8.3 Graph ax2+bx+c with work

a) Find the axis of symmetry

$$f(x) = 2x^2 + 8x - 1$$
$$\underset{ax^2 + bx + c}{}$$

$x = \dfrac{-b}{2a}$ $a = 2$ $b = 8$ $c = -1$

$x = \dfrac{-8}{2(2)} = -2$

$\boxed{x = -2}$

b) Find the vertex $\boxed{(-2, -9)}$

$f(-2) = 2(-2)^2 + 8(-2) - 1$

$= 8 - 16 - 1$

$= -9$
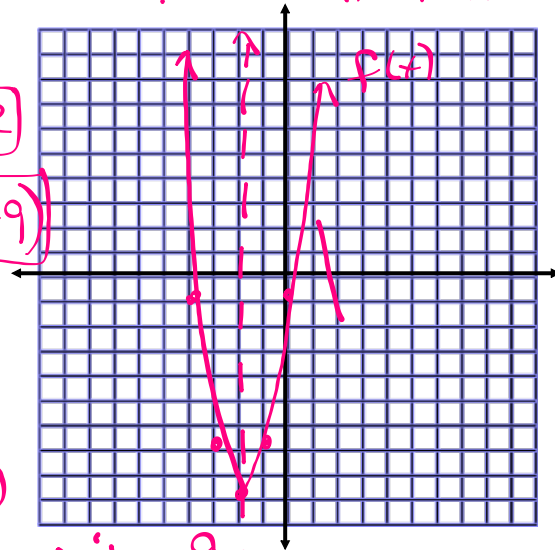
c) Graph the function

x		f(x)
-4	$2(-4)^2 + 8(-4) - 1$	-1
-3	$2(-3)^2 + 8(-3) - 1$	-7
-2		-9
-1		-7
0		-1

Min; $-9 \downarrow$

D: $\mathbb{R}$

R: $y \geq -9$

Feb 23-5:10 PM

Graph the function and describe the domain and range.
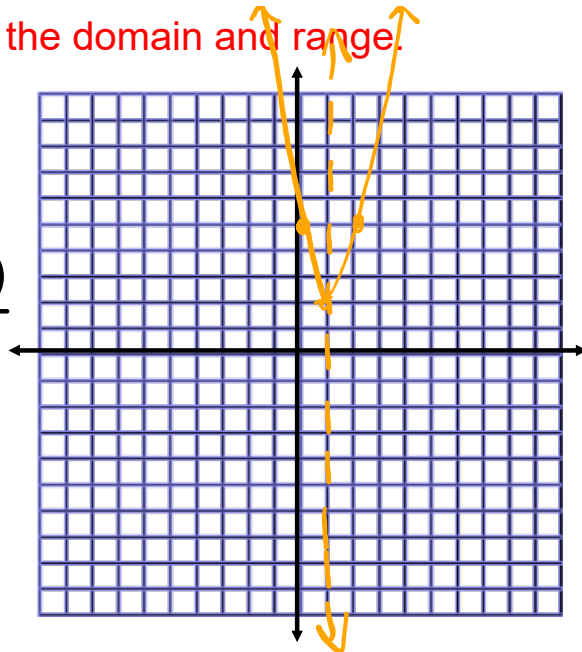
$$f(x) = 3x^2 - 6x + \boxed{5}$$

$\dfrac{-b}{2a} = \dfrac{-(-6)}{2(3)} = \dfrac{6}{6} = 1$

x	$f(x) = 3x^2 - 6x + 5$	f(x)
-1	$3(-1)^2 - 6(-1) + 5$	14
0	$3(0)^2 - 6(0) + 5$	5
1	$3(1)^2 - 6(1) + 5$	2
2		5
3		14

D: $\mathbb{R}$

R: $y \geq 2$

Min; 2

Feb 23-5:16 PM

3

SOLUTION

Step 1 Find and graph the axis of symmetry.

$$x = -\frac{b}{2a} = -\frac{(-6)}{2(3)} = 1 \qquad \text{Substitute and simplify.}$$

Step 2 Find and plot the vertex.

The axis of symmetry is $x = 1$, so the x-coordinate of the vertex is 1. Use the function to find the y-coordinate of the vertex.

$$f(x) = 3x^2 - 6x + 5 \qquad \text{Write the function.}$$
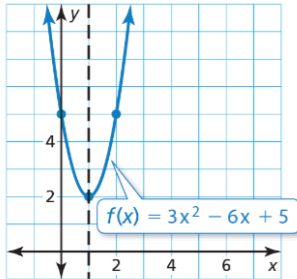$$f(1) = 3(1)^2 - 6(1) + 5 \qquad \text{Substitute 1 for } x.$$
$$= 2 \qquad \text{Simplify.}$$

So, the vertex is (1, 2).

Step 3 Use the y-intercept to find two more points on the graph.

Because $c = 5$, the y-intercept is 5. So, (0, 5) lies on the graph. Because the axis of symmetry is $x = 1$, the point (2, 5) also lies on the graph.

Step 4 Draw a smooth curve through the points.

$f(x) = 3x^2 - 6x + 5$

Feb 23-5:17 PM

Tell whether the function $f(x) = -4x^2 - 24x - 19$ has a minimum value or maximum value. Then find the value.

$a = -4$

$\boxed{\text{Max; } 17}$

$$-\frac{b}{2a} = \frac{-(-24)}{2(-4)} = \frac{24}{-8} = -3$$

$$f(-3) = -4(-3)^2 - 24(-3) - 19$$

$$f(-3) = 17$$

$$\rightarrow (-3, 17)$$

-4(-3)²-24(-3)-19
17

Tell whether the function $f(x) = 5x^2 + 15x - 10$ has a minimum value or a maximum value. Then find the value.

$a = 5$

$\boxed{\text{Min; } -21.25}$

$$-\frac{b}{2a} = \frac{-15}{2(5)} = \frac{-15}{10} = \frac{-3}{2}$$

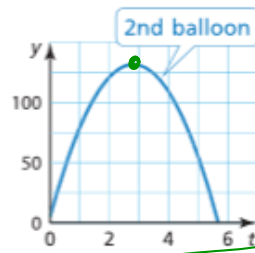$$f\left(-\frac{3}{2}\right) = 5\left(-\frac{3}{2}\right)^2 + 15\left(-\frac{3}{2}\right) - 10$$

$$= -21.25$$

5(-3/2)²+15(-3/2)-10
-21.25

Feb 23-5:10 PM

4

A group of friends is launching water balloons. The function $f(t) = -16t^2 + 80t + 5$ represents the height (in feet) of the first water balloon t seconds after it is launched. The height of the second water balloon t seconds after it is launched is shown in the graph. Which water balloon went higher?

2nd balloon

$y > 125$

$-\dfrac{b}{2a} = \dfrac{-80}{2(-16)} = \dfrac{-80}{-32} = \dfrac{5}{2}$

$f\left(\dfrac{5}{2}\right) = -16\left(\dfrac{5}{2}\right)^2 + 80\left(\dfrac{5}{2}\right) + 5$

$f\left(\dfrac{5}{2}\right) = 105$

2nd balloon went higher.

```
              -21.25
-16(5/2)²+80(5/0
             Error
-16(5/2)²+80(5/2
                105
```

Feb 23-5:23 PM

8.3 Graph $f(x) = ax^2 + bx + c$

Assign

p 436

A: 2-8(e), 12-16(e), 20, 24-28(e), 32, 38-42(e), 52

B: 2 - 10(e), 14, 16, 20, 22, 24, 28, 32, 38, 40, 52

C: 4 - 28(e), 52

Feb 23-5:25 PM