8.4 Graph $f(x) = a(x-h)^2 + k$

Essential Question:

How can you describe the graph of $f(x) = a(x - h)^2$?

What You Will Learn:

- Identify even and odd functions.
- Graph quadratic functions of the form $f(x) = a(x - h)^2$.
- Graph quadratic functions of the form $f(x) = a(x - h)^2 + k$.
- Model real-life problems using $f(x) = a(x - h)^2 + k$.

Core Vocabulary:

even function
odd function
vertex form (of a quadratic function
reflection

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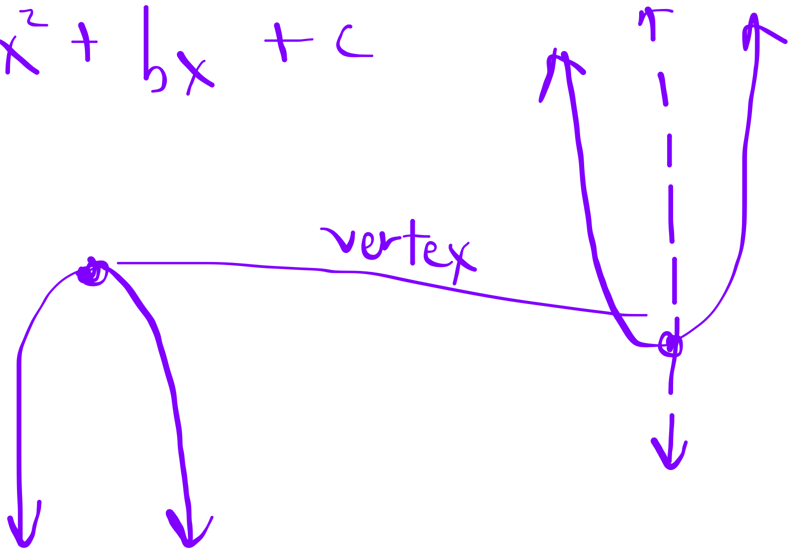
$y = ax^2 + bx + c$



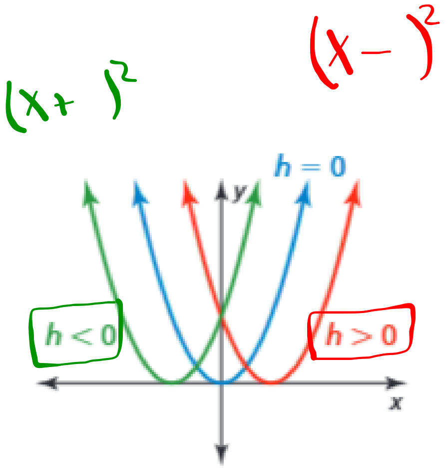
Graphing $f(x) = a(x - h)^2$

Core Concept

Graphing $f(x) = a(x - h)^2$

- When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units right of the graph of $f(x) = ax^2$.
- When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation $|h|$ units left of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = a(x - h)^2$ is $(h, 0)$, and the axis of symmetry is $x = h$.



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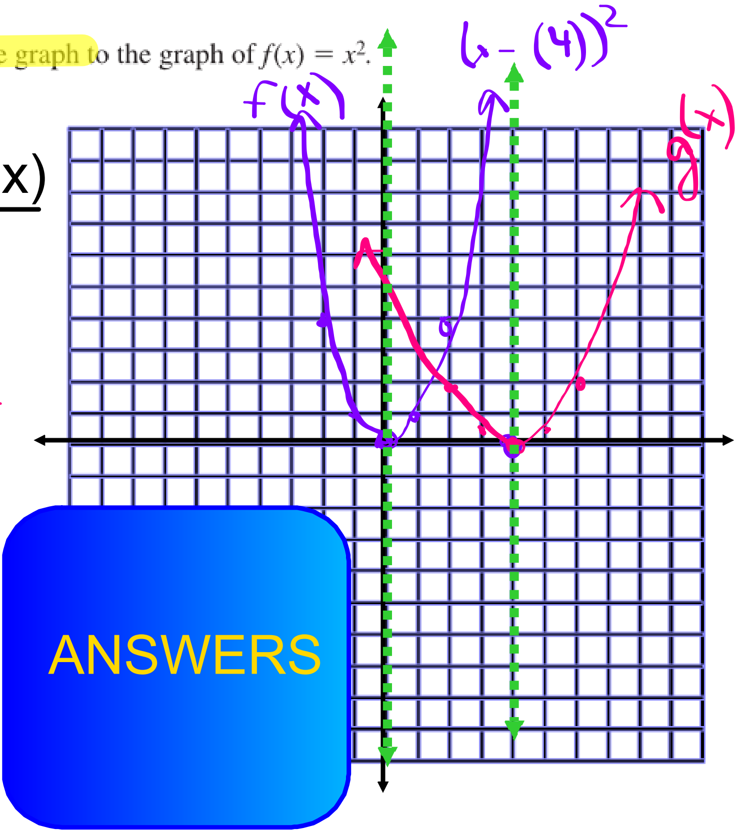
Graph $g(x) = \frac{1}{2}(x - 4)^2$. Compare the graph to the graph of $f(x) = x^2$.

$a = \frac{1}{2}$ $h = 4$ VC $\frac{1}{2}$

x	$\frac{1}{2}(x - 4)^2$	g(x)
2	$\frac{1}{2}(2-4)^2$	2
3	$\frac{1}{2}(3-4)^2$	$\frac{1}{2}$
4	$\frac{1}{2}(4-4)^2$	0
5		$\frac{1}{2}$
6		2

Function	$f(x) = x^2$	$f(x) = \frac{1}{2}(x - 4)^2$
Axis of symmetry	$x = 0$	$x = 4$
Vertex	(0, 0)	(4, 0)
Type of shrink or stretch (factor)	—	vertical shrink $\left(\frac{1}{2}\right)$
Translation	—	4 units right
Reflection	—	—



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8.4 Graph y a(x-h)2 + k DAY ONE with work

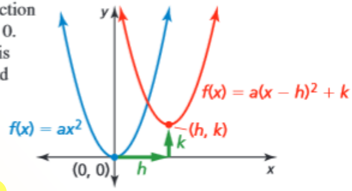
Graphing $f(x) = a(x - h)^2 + k$

Core Concept

Graphing $f(x) = a(x - h)^2 + k$

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The graph of $f(x) = a(x - h)^2 + k$ is a translation h units horizontally and k units vertically of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = a(x - h)^2 + k$ is (h, k), and the axis of symmetry is $x = h$.



Examples: Assume a = 1 $y = a(x-h)^2 + k$

Write the equation of a parabola with vertex at:

1) (2, 3)   2) (-1, -4)   3) (-3, 0)

1) $y = (x-2)^2 + 3$ 2) $y = (x+1)^2 - 4$ 3) $x - (-3)$, $y = (x+3)^2$

Give the vertex of the parabola:

1) $f(x) = 3(x - 5)^2 + 7$ (5, 7)

2) $y = 0.5(x + 3)^2 - 8$ (-3, -8) $(x - (-3))^2$

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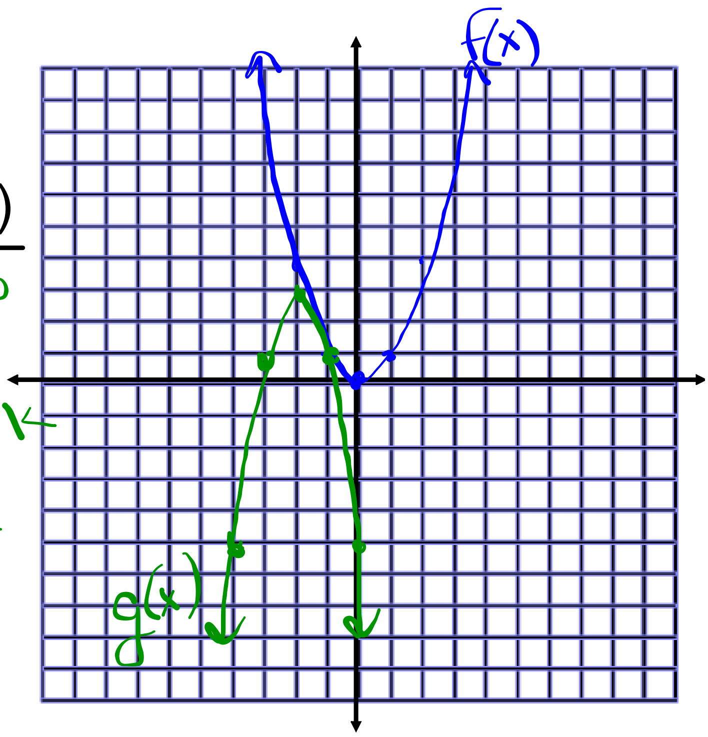
Graph $g(x) = -2(x + 2)^2 + 3$. Compare the graph to the graph of $f(x) = x^2$.

Transformations:

R_x, VS2, ← 2u, ↑ 3u

x	$-2(x+2)^2+3$	g(x)
0	$-2(0+2)^2+3$	-5
-1	$-2(-1+2)^2+3$	1
h -2	$-2(-2+2)^2+3$	3 k
-3		1
-4		-5



Domain: ℝ

Range: y < 3

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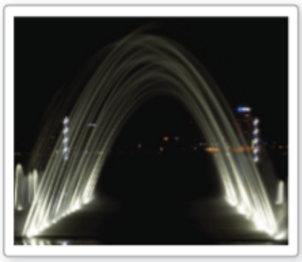
8.4 Graph y a(x-h)2 + k DAY ONE with work

Real-life example



Water fountains are usually designed to give a specific visual effect. For example the water fountain shown consists of streams of water that are shaped like parabolas. Notice how the streams are designed to land on the underwater spotlights. Write and graph a quadratic function that models the path of a stream of water with a maximum height of 5 feet, represented by a vertex of (3, 5), landing on a spotlight 6 feet from the water jet represented by (6,0).

x y

$y = a(x-h)^2 + k$ $h = 3$ $k = 5$

$0 = a(6-3)^2 + 5$

$0 = a(3)^2 + 5$

$0 = 9a + 5$

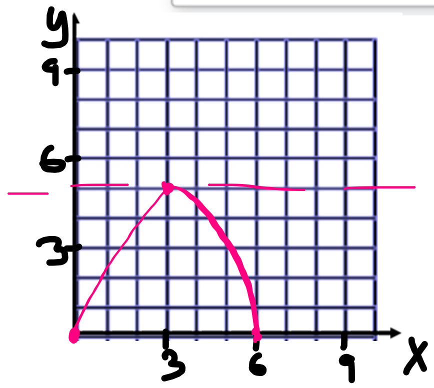
$\frac{-5}{9} = \frac{9a}{9}$

$a = -\frac{5}{9}$

$h = 3$

$k = 5$

$y = -\frac{5}{9}(x-3)^2 + 5$



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SOLUTION

1. **Understand the Problem** You know the vertex and another point on the graph that represents the parabolic path. You are asked to write and graph a quadratic function that models the path.

2. **Make a Plan** Use the given points and the vertex form to write a quadratic function. Then graph the function.

3. **Solve the Problem**
Use the vertex form, vertex (3, 5), and point (6, 0) to find the value of a.

$f(x) = a(x-h)^2 + k$	Write the vertex form of a quadratic function.
$f(x) = a(x-3)^2 + 5$	Substitute 3 for h and 5 for k.
$0 = a(6-3)^2 + 5$	Substitute 6 for x and 0 for $f(x)$.
$0 = 9a + 5$	Simplify.
$-\frac{5}{9} = a$	Solve for a.

So, $f(x) = -\frac{5}{9}(x-3)^2 + 5$ models the path of a stream of water. Now graph the function.

Step 1 Graph the axis of symmetry. Because $h = 3$, graph $x = 3$.

Step 2 Plot the vertex, (3, 5).

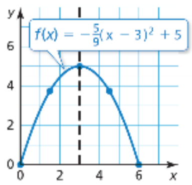
Step 3 Find and plot two more points on the graph. Because the x-axis represents the water surface, the graph should only contain points with nonnegative values of $f(x)$. You know that (6, 0) is on the graph. To find another point, choose an x-value between $x = 3$ and $x = 6$. Then find the corresponding value of $f(x)$.

$f(4.5) = -\frac{5}{9}(4.5-3)^2 + 5 = 3.75$

So, plot (6, 0) and (4.5, 3.75).

Step 4 Reflect the points plotted in Step 3 in the axis of symmetry. So, plot (0, 0) and (1.5, 3.75).

Step 5 Draw a smooth curve through the points.



4. **Look Back** Use a graphing calculator to graph $f(x) = -\frac{5}{9}(x-3)^2 + 5$. Use the *maximum* feature to verify that the maximum value is 5. Then use the *zero* feature to verify that $x = 6$ is a zero of the function.

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8.4 WS A

#'s: 6 - 8, 10, 11, 12, 13, 15, 16, 19