8.4 Variation & Problem Solving DAY ONE

OBJECTIVE 1: Solving Problems Involving Direct Variation

Direct variation deals with a constant of variation that is proportional. Typically the problem will read, "C varies directly as r, or y varies directly as x. This produces the function **y = kx** where k is the constant of variation. This variation will produce a linear graph with k as the slope that will pass through the origin.

Direct Variation

y varies directly as *x*, or *y* is directly proportional to *x*, if there is a nonzero constant *k* such that

$$y = kx$$

The number *k* is called the constant of variation or the constant of proportionality.

Example 1: Suppose that y varies directly as x. If y is 5 when x is 30, find the constant of variation and the direct variation equation.

k

$y = kx$

$y = 5 \Rightarrow x = 30$

$\frac{5}{30} = \frac{k(\cancel{30})}{\cancel{30}}$

$\boxed{k = \frac{1}{6}}$ $\boxed{y = \frac{1}{6}x}$ $\boxed{y = \frac{x}{6}}$

Practice 1: What if y is 20 when x is 15?

$y = 20 \quad x = 15$

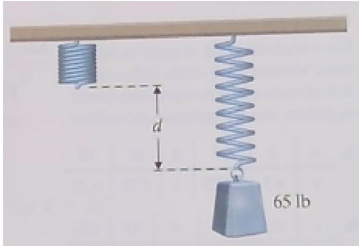
$y = kx$

$\frac{20}{15} = \frac{k(\cancel{15})}{\cancel{15}}$

$\boxed{y = \frac{4}{3}x}$ $\boxed{\frac{4}{3} = k}$

Example 2: Using Direct Variation and Hooke's Law

Hooke's law states that the distance a spring stretches is directly proportional to the weight attached to the spring. If a 40-pound weight attached to the spring stretches the spring 5 inches, find the distance that a 65-pound weight attached to the spring stretches the spring.

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65/8
                8.125
Ans▸Frac
                 65/8
```

$y = kx$

$\frac{5}{40} = \frac{k(40)}{40}$

$\boxed{\frac{1}{8} = k}$

$\boxed{y = \frac{1}{8}x}$

$y = \frac{1}{8}(65) = \frac{65}{8} \approx \boxed{8.125 \text{ in}}$

Practice 2:

Use Hooke's law as stated in Example 2. If a 36-pound weight attached to a spring stretches the spring 9 inches, find the distance that a 75-pound weight attached to the spring stretches the spring.

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75/4
                18.75
Ans▸Frac
                 75/4
```

$y = kx$

$\frac{9}{36} = \frac{k(36)}{36}$

$\boxed{\frac{1}{4} = k}$

$\boxed{y = \frac{1}{4}x}$

$y = \frac{1}{4}(75) = \frac{75}{4}$

$\boxed{\approx 18.75 \text{ in}}$

OBJECTIVE 2: Solving Problems Involving Inverse Variation

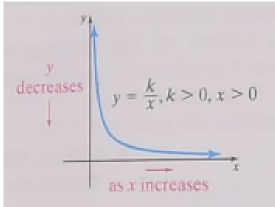
When y is proportional to the reciprocal of another variable x, then we say "y varies inversely as x" or inversely proportional.

Inverse Variation

y varies inversely as x, or y is inversely proportional to x, if there is a nonzero constant k such that

$$y = \frac{k}{x}$$

The number k is called the **constant of variation** or the **constant of proportionality.**



Example 3: Suppose the u varies inversely as w. If u is 3 when w is 5, find the constant of variation and the inverse variation equation.

$\boxed{u = \frac{15}{w}}$

$u = \frac{k}{w}$

$5 \cdot 3 = \frac{k}{\cancel{5}} \cdot \cancel{5}$

$\boxed{15 = k}$

$w = 5$
$u = 3$
$k = ?$

Practice 3:
Now b varies inversely as a. If b is 5 when a is 9, find the constant of variation and the inverse variation equation.

$b = \frac{k}{a}$

$9 \cdot 5 = \frac{k}{\cancel{9}} \cdot \cancel{9}$

$\boxed{45 = k}$

$b = 5$
$a = 9$
$k = ?$

$\boxed{b = \frac{45}{a}}$

Example 4: Using Inverse Variation and Boyle's Law

Boyle's law says that if the temperature stays the same, the pressure P of a gas is inversely proportional to the volume V. If a cylinder in a steam engine has a pressure of 960 kilopascals when the volume is 1.4 cubic meters, find the pressure when the volume increases to 2.5 cubic meters.

$y = \frac{k}{x} \Rightarrow P = \frac{k}{V}$

$P = 960$

$V = 1.4$

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960*1.4
              1344
1344/2.5
             537.6
```

$1.4 \cdot 960 = \frac{k}{\cancel{1.4}} \cdot \cancel{1.4}$

$\boxed{1344 = k}$

$P = \frac{1344}{V}$

$P = \frac{1344}{2.5}$

$\boxed{\approx 537.6 \text{ kilopascals}}$

Practice 4:

Use Boyle's law as stated in Example 4. If P = 350 kilopascals when V = 2.8 cubic meters, find the pressure when the volume decreases to 1.5 cubic meters.

$P = \frac{k}{V}$

$P = 350$

$V = 2.8$

```
2.8*350
                 980
980/1.5
          653.3333333
Ans▸Frac
                1960/3
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$2.8 \cdot 350 = \frac{k}{\cancel{2.8}} \cdot \cancel{2.8}$

$\boxed{980 = k}$ $\boxed{P = \frac{980}{V}}$

$P = \frac{980}{1.5}$ $\boxed{\approx 653.\overline{3} \text{ kilopascals}}$

8.4 DAY ONE HW Assignment

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