

9.4: Solving Quadratic Equations by "Completing the Square"

Essential Question

How can you use "completing the square" to solve a quadratic equation?

Essential Question

Core Concept

$$\square = \left(\frac{b}{2}\right)^2$$

Completing the Square

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1 Find one-half of b , the coefficient of x .

Step 2 Square the result from Step 1.

Step 3 Add the result from Step 2 to $x^2 + bx$.

Factor the resulting expression as the square of a binomial.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Core Concept

9.4 Completing the Square with work

Examples:

Complete the square for each expression. Then **factor** the trinomial.

a. $x^2 + 18x + \boxed{81}$

$$\begin{aligned} \square &= \left(\frac{b}{2}\right)^2 \\ &= \left(\frac{18}{2}\right)^2 \\ &= (9)^2 \\ &= \boxed{81} \end{aligned}$$

$x^2 + 18x + 81$
 $(x+9)^2$

b. $x^2 - 17x + \boxed{\frac{289}{4}}$

$$\begin{aligned} \square &= \left(\frac{b}{2}\right)^2 = \left(\frac{-17}{2}\right)^2 \\ &= \frac{289}{4} \end{aligned}$$

$\left(x - \frac{17}{2}\right)^2$

Example 1

YOUR TURN:

Complete the square for the expression. Then factor the trinomial.

1. $x^2 + 10x$

$$\begin{aligned} &\boxed{25} \\ &\boxed{(x+5)^2} \end{aligned}$$

2. $x^2 - 4x$

$$\begin{aligned} &\boxed{4} \\ &\boxed{(x-2)^2} \end{aligned}$$

3. $x^2 + 7x$

$$\begin{aligned} &\boxed{\frac{49}{4}} \\ &\boxed{\left(x + \frac{7}{2}\right)^2} \end{aligned}$$

9.4 Completing the Square with work

EXAMPLE:

Solve $x^2 - 18x = -17$ by completing the square.

$$x^2 - 18x + \boxed{81} = -17 + \boxed{81}$$

$$\square = \left(\frac{-18}{2}\right)^2 = \boxed{81}$$

$$\sqrt{(x-9)^2} = \sqrt{64}$$

$$\boxed{x = 17, 1}$$

$$\begin{array}{l} x-9 = \pm 8 \\ +9 \quad +9 \end{array}$$

$$x = 9 \pm 8$$

Example 2

EXAMPLE:

Solve $2x^2 + 12x - 10 = 0$ by completing the square.

$$x^2 + 6x - 5 = 0$$

$$\square = \left(\frac{6}{2}\right)^2 = 9$$

$$x^2 + 6x + \boxed{9} = 5 + \boxed{9}$$

$$\sqrt{(x+3)^2} = \sqrt{14}$$

$$\boxed{x = -3 \pm \sqrt{14}}$$

$$\begin{array}{l} x+3 = \pm \sqrt{14} \\ -3 \quad -3 \end{array}$$

Example 3

9.4 Completing the Square with work

YOUR TURN:

Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

4. $x^2 - 2x = 3$

$$x^2 - 2x + \boxed{1} = 3 + \boxed{1}$$

$$\left(\frac{-2}{2}\right)^2 = 1$$

$$\sqrt{(x-1)^2} = \sqrt{4}$$

$$x-1 = \pm 2$$

$$x = 1 \pm 2$$

$$\boxed{x = 3, -1}$$

5. $m^2 + 12m = -8$

$$m^2 + 12m + \boxed{36} = -8 + \boxed{36}$$

$$\left(\frac{12}{2}\right)^2 = 36$$

$$\sqrt{(m+6)^2} = \sqrt{28} \quad \text{④}$$

$$m+6 = \pm 2\sqrt{7}$$

$$\boxed{m = -6 \pm 2\sqrt{7}}$$

6. $3g^2 - 24g + 27 = 0$

$$g^2 - 8g + 9 = 0$$

$$g^2 - 8g + \boxed{16} = -9 + \boxed{16}$$

$$\left(\frac{-8}{2}\right)^2 = 16$$

$$\sqrt{(g-4)^2} = \sqrt{7}$$

$$g-4 = \pm\sqrt{7}$$

$$\boxed{g = 4 \pm \sqrt{7}}$$

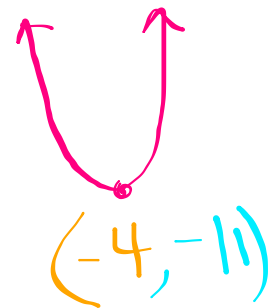
Monitoring Progress 4-6

Example:

Find the minimum value of $y = x^2 + 8x + 5$.

$$a > 0 \quad \cup \quad \text{min}$$

$$a < 0 \quad \cap \quad \text{max}$$



$$\frac{-b}{2a} = \frac{-8}{2(1)} = \frac{-8}{2} = -4$$

$$f(-4) = (-4)^2 + 8(-4) + 5 = 16 - 32 + 5 = -11$$

$$\boxed{-11}$$

$$(-4)^2 + 8(-4) + 5 = -11$$

Example 4

9.4 Completing the Square with work

Example:

Find the maximum value of $y = -x^2 + 4x + 2$.

$$\frac{+4}{2(-1)} = -2$$

$$-(-2)^2 + 4(-2) + 2$$

$$-4 - 8 + 2 = \boxed{-10}$$

Example 5

YOUR TURN:

Determine whether the quadratic function has a maximum or minimum value. Then find the value.

7. $y = -x^2 - 4x + 4$

$$\frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$$

$$-(-2)^2 - 4(-2) + 4$$
$$-4 + 8 + 4$$

$$\boxed{8} \quad \boxed{\text{max}}$$

8. $y = x^2 + 12x + 40$

$$\frac{-12}{2(1)} = -6$$

$$(-6)^2 + 12(-6) + 40$$
$$36 - 72 + 40$$

$$\boxed{4} \quad \boxed{\text{min}}$$

9. $y = x^2 - 2x - 2$

$$\frac{-(-2)}{2(1)} = \frac{2}{2} = 1$$

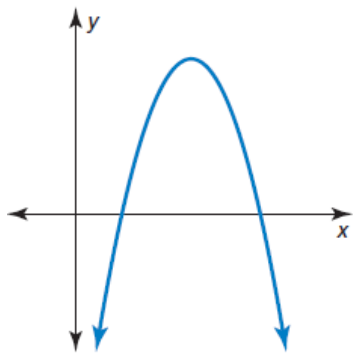
$$(1)^2 - 2(1) - 2$$

$$1 - 2 - 2$$

$$\boxed{-3} \quad \boxed{\text{min}}$$

9.4 Completing the Square with work

Which of the functions could be represented by the graph? Explain.



~~$f(x) = -\frac{1}{2}(x + 4)^2 + 8$~~

$(-4, 8)$

$g(x) = -(x - 5)^2 + 9$

$(5, 9)$

~~$m(x) = (x - 3)(x - 12)$~~

$p(x) = -(x - 2)(x - 8)$

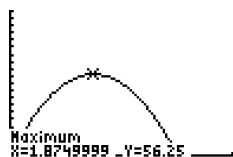
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Example 6

REAL-WORLD APPLICATION

The function $y = -16x^2 + 60x$ represents the height y (in feet) of a model rocket x seconds after it is launched.

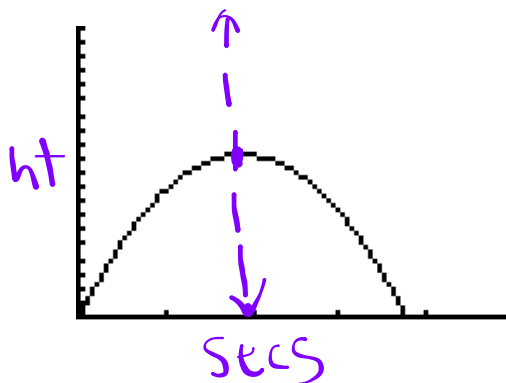
(a) Find the maximum height of the rocket.



56.25 ft

(b) Find and interpret the axis of symmetry.

$x = 1.875$

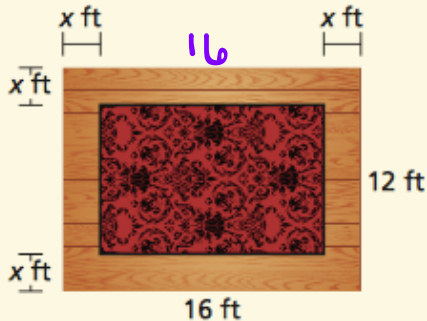


Example 7

9.4 Completing the Square with work

Practice:

You decide to put a rug on a floor. You want the rug to cover 100 square feet and to have a uniform border of floor visible, as shown. Find the width of the border to the nearest inch.



$$(16+2x)(12+2x)$$

$$192 + 24x + 32x + 4x^2$$

$$192 + 56x + 4x^2$$

$$100 = 48 + 14x + x^2$$

$$x^2 + 14x + \boxed{49} = 100 + \boxed{49}$$

$$\left(\frac{14}{2}\right)^2 = 49$$

$$\frac{-7 + \sqrt{149}}{5.206555616}$$

$$5\text{ft} \approx \boxed{60\text{ in}}$$

$$\sqrt{(x+7)^2} = \sqrt{49}$$

$$x+7 = \pm\sqrt{149}$$

$$x = -7 \pm \sqrt{149}$$

Example 8

HW Assignment:

9.4 WS's

WS WB: 5, 6, 10, 11, 17, 18, 23, 24, 25 (9?s)

WS A: evens (11?s)

20 total questions for tonight!