9.5: Solving Quadratic Equations using the "Quadratic Formula"

Essential Question

How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

Essential Question

The following steps show a method of solving $ax^2 + bx + c = 0$.

Explain what was done in each step.

$$ax^2 + bx + c = 0$$ ← original

$$4a^2x^2 + 4abx + 4ac = 0$$ ← Multiply 4a

$$4a^2x^2 + 4abx + 4ac + b^2 = b^2$$ ← $+b^2$ both sides

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$ ← Subtract 4ac

$$(2ax + b)^2 = b^2 - 4ac$$ ← Factor

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$ ← Sq. Rt. Method

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$ ← Subtract b

Quadratic Formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← Divide by 2a

Exploration 1

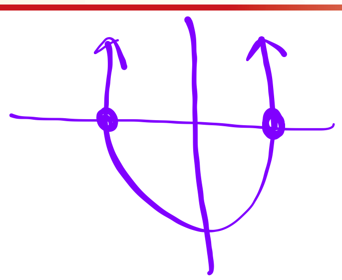
Core Concept

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

where $a \neq 0$ and $b^2 - 4ac \geq 0$.



Core Concept

Example:

Solve $12x^2 - 4x - 5 = 0$ using the Quadratic Formula. State what a, b, and c you are using to show work.

$a = 12$ $b = -4$ $c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$(-4)^2 - 4(12)(-5)$

256

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(12)(-5)}}{2(12)}$$

$$x = \frac{4 \pm \sqrt{256}}{24} = \frac{4 \pm 16}{24}$$

$\frac{4+16}{24} = \frac{20}{24} = \frac{5}{6}$

$\frac{4-16}{24} = \frac{-12}{24} = -\frac{1}{2}$

$x = \frac{5}{6}, -\frac{1}{2}$

Example 1

YOUR TURN:

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

1. $x^2 - 6x + 5 = 0$

$a = 1 \quad b = -6 \quad c = 5$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$

$= \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2}$

$\frac{6+4}{2} = 5 \quad \frac{6-4}{2} = 1 \quad \boxed{x = 1, 5}$

2. $\frac{1}{2}x^2 + x - 10 = 0$

$a = \frac{1}{2} \quad b = 1 \quad c = -10$

$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(\frac{1}{2})(-10)}}{2(\frac{1}{2})}$

$= \boxed{-1 \pm \sqrt{21}}$

3. $-3x^2 + 2x + 7 = 0$

$a = -3 \quad b = 2 \quad c = 7$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(-3)(7)}}{2(-3)}$

$= \frac{-2 \pm \sqrt{88}}{-6} \quad \frac{-2 \pm 2\sqrt{22}}{-6}$

$\boxed{x = \frac{-1 \pm \sqrt{22}}{-3} \text{ OR } \frac{1 \pm \sqrt{22}}{3}}$

4. $4x^2 - 4x = -1$

$4x^2 - 4x + 1 = 0$

$a = 4 \quad b = -4 \quad c = 1$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$

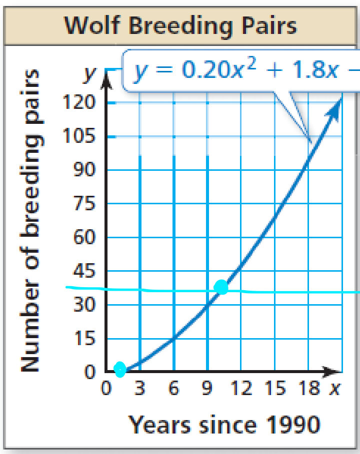
$= \frac{4 \pm \sqrt{0}}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$

Monitoring Progress 1-4

REAL-WORLD APPLICATION:

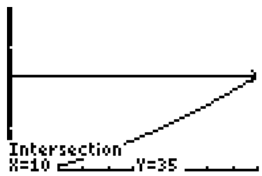
The number y of Northern Rocky Mountain wolf breeding pairs x years since 1990 can be modeled by the function $y = 0.20x^2 + 1.8x - 3$. When were there about 35 breeding pairs?

$a = 0.20 \quad b = 1.8 \quad \boxed{c = -3}$



$x = \frac{-(1.8) \pm \sqrt{(1.8)^2 - 4(0.20)(-3)}}{2(0.20)}$

$= \frac{-1.8 \pm \sqrt{5.64}}{0.4}$

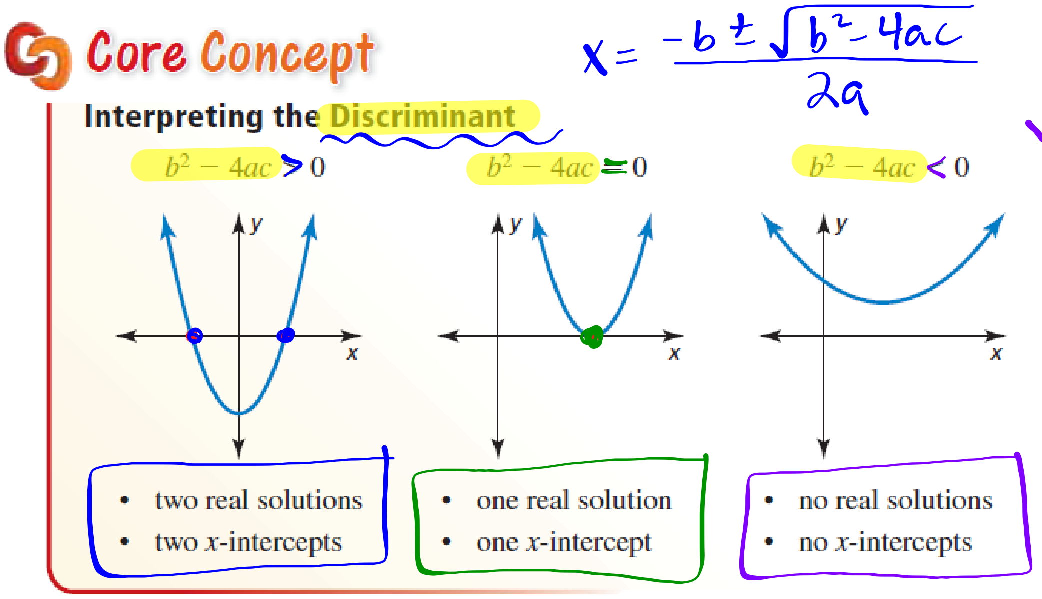


$x \approx 1.437, ~~-10.437~~$

10 years after 1990.

Example 2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Core Concept

Example:

$b^2 - 4ac$

a. Determine the number of real solutions of $2x^2 - 3x - 1 = 0$.

$a = 2 \quad b = -3 \quad c = -1$

$(-3)^2 - 4(2)(-1)$

$9 + 8 = 17 > 0$

2 real sol.

b. Determine the number of real solutions of $x^2 + 4 = x$.

$x^2 - x + 4 = 0$

$a = 1 \quad b = -1 \quad c = 4$

$(-1)^2 - 4(1)(4)$

$= 1 - 16 = -15 < 0$

no real sol.

Example 3

YOUR TURN:

Determine the number of real solutions of the equation.

7. $-x^2 + 4x - 4 = 0$

$a = -1 \quad b = 4 \quad c = -4$

$= (4)^2 - 4(-1)(-4)$

$= 16 - 16 = 0$

1 real sol.

8. $6x^2 + 2x = -1$

$+1 \quad +1$

$6x^2 + 2x + 1 = 0$

$a = 6 \quad b = 2 \quad c = 1$

$= (2)^2 - 4(6)(1)$

$= 4 - 24 = -20 < 0$

no real sol.

9. $\frac{1}{2}x^2 = 7x - 1$

$\frac{1}{2}x^2 - 7x + 1 = 0$

$a = \frac{1}{2} \quad b = -7 \quad c = 1$

$= (-7)^2 - 4(\frac{1}{2})(1)$

$= 49 - 2 = 47 > 0$

2 real sol.

Monitoring Progress 7-9

EXAMPLE:

$b^2 - 4ac$

Find the number of x-intercepts of the graph of $y = 2x^2 - 4x - 3$.

- roots
- zeros
- solutions

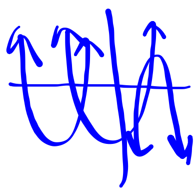
$a = 2 \quad b = -4 \quad c = -3$

$(-4)^2 - 4(2)(-3)$

$= 16 + 24$

$= 40 > 0$

2 x-int.



Example 4

9.5 Quadratic Formula with work

YOUR TURN:

$b^2 - 4ac$

Find the number of x-intercepts of the graph of the function.

10. $y = -x^2 + x - 6$

$a=-1 \quad b=1 \quad c=-6$

$(1)^2 - 4(-1)(-6)$

$1 - 24$

$-23 < 0$

no x-int

11. $y = x^2 - x$

$a=1 \quad b=-1 \quad c=0$

$(-1)^2 - 4(1)(0)$

$1 - 0 = 1 > 0$

2 x-int.

12. $f(x) = x^2 + 12x + 36$

$a=1 \quad b=12 \quad c=36$

$(12)^2 - 4(1)(36)$

$144 - 144$

$0 = 0$

1 x-int.

Monitoring Progress 10-12

Core Concept

Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring *(Lessons 7.5–7.8)*	• Straightforward when the equation can be factored easily	• Some equations are not factorable.
Graphing *(Lesson 9.2)*	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots *(Lesson 9.3)*	• Use to solve equations of the form $x^2 = d$. $b=0$	• Can only be used for certain equations
Completing the Square *(Lesson 9.4)*	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula *(Lesson 9.5)*	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do calculations • formula

Core Concept

9.5 Quadratic Formula with work

Practice:

Solve the equation using any method. Explain your choice of method.

a. $x^2 - 4x - 7 = 0$

$+7 \quad +7$

$x^2 - 4x + 4 = 7 + 4$

$D = \left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2$

$\sqrt{(x-2)^2} = \sqrt{11}$

$x - 2 = \pm\sqrt{11}$

$x = 2 \pm \sqrt{11}$

b. $\sqrt{(x - 2)^2} = \sqrt{64}$

$x - 2 = \pm 8$

$+2 \quad +2$

$x = 2 \pm 8$

$2 + 8 = 10$

$2 - 8 = -6$

$x = -6, 10$

c. $10x^2 + 7x - 12 = 0$

$a = 10 \quad b = 7 \quad c = -12$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(10)(-12)}}{2(10)}$$

(7)²-4(10)(-12)
529

$$= \frac{-7 \pm \sqrt{529}}{20}$$

$$= \frac{-7 \pm 23}{20}$$

$\frac{-7 + 23}{20} = \frac{16}{20} = \frac{4}{5}$ ($\div 4$)

$\frac{-7 - 23}{20} = \frac{-30}{20} = -\frac{3}{2}$

$x = \frac{4}{5}, -\frac{3}{2}$

Example 5

HW Assignment 9.5

WS from the WB

odds