9.7

Law of Sines and Law of Cosines
&
Area of a Triangle

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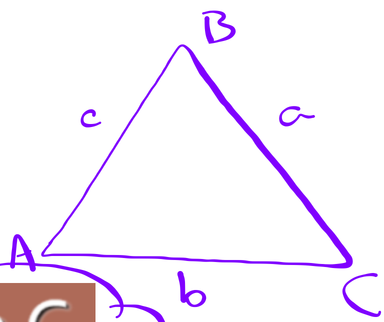
Law of Sines

AAS, ASA, or SSA

sine < 1

For any triangle with angles A, B, & C, and sides of lengths a, b, & c.

- a is the opposite of angle A
- b is the opposite of angle B
- c is the opposite of angle C

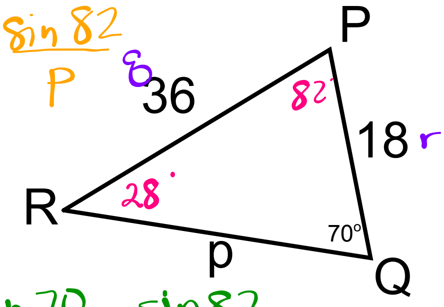


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example:

Solve the triangle. Round decimals to the nearest tenth.

$$\frac{\sin Q}{q} = \frac{\sin R}{r} \qquad \frac{\sin Q}{q} = \frac{\sin 82}{p}$$



$$\frac{\sin 70}{36} = \frac{\sin R}{18}$$

$$\cancel{36}\frac{(\sin R)}{\cancel{36}} = \frac{18(\sin 70)}{36}$$

$$\sin R = \frac{18(\sin 70)}{36}$$

$$R = \sin^{-1}\left(\frac{18\sin 70}{36}\right)$$

sin⁻¹(18sin(70)/▸
28.02432067

180-70-28
82

$$\boxed{m\angle R = 28°}$$

$$\frac{\sin 70}{36} = \frac{\sin 82}{p}$$

$$\frac{p(\cancel{\sin 70})}{\cancel{\sin 70}} = \frac{36(\sin 82)}{(\sin 70)}$$

$$p = \frac{36(\sin 82)}{\sin 70}$$

(36sin(82))/(sin(7▸)
37.93756563

$$\boxed{p \approx 37.9 \text{ u}}$$

Law of Cosine

SSS or SAS

Three versions:

- $c^2 = a^2 + b^2 - 2ab(\cos C)$
- $b^2 = a^2 + c^2 - 2ac(\cos B)$
- $a^2 = b^2 + c^2 - 2bc(\cos A)$

*** Notice that the side at the beginning & the angle at the end match up!***

Example:

Solve the triangle. Round decimal answers to the nearest tenths.

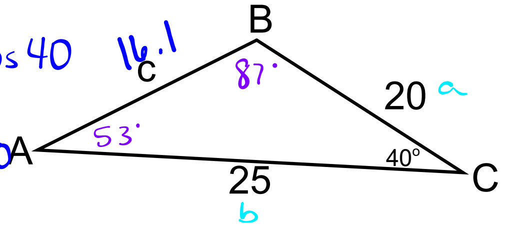
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 20^2 + 25^2 - 2(20)(25)\cos 40$ 16.1

$c = \sqrt{400 + 625 - 1000\cos 40}$

√400+625-1000cos
16.0920961

$\boxed{c \approx 16.1}$



$a^2 = b^2 + c^2 - 2bc\cos A$

$20^2 = 25^2 + (16.1)^2 - 2(25)(16.1)\cos A$

$400 = 625 + 259.21 - 805\cos A$

$-484.21 = -805\cos A$

$0.602 = \cos A$

$A = \cos^{-1}(0.602)$

$\boxed{m\angle A = 53°}$

cos⁻¹(0.602)
52.98672822

Area of a Triangle using Trig

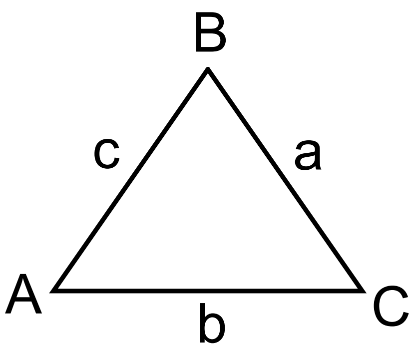
$A = \frac{1}{2}bh$

Three ways:

$$A = \frac{1}{2}bc\sin A$$

$$A = \frac{1}{2}ac\sin B$$

$$A = \frac{1}{2}ab\sin C$$



Example:

Find the area of the triangle. Round your answer to the nearest tenth.

$A = \frac{1}{2}(12)(15)\sin 110°$

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(1/2)(12)(15)(s▸
     84.57233587
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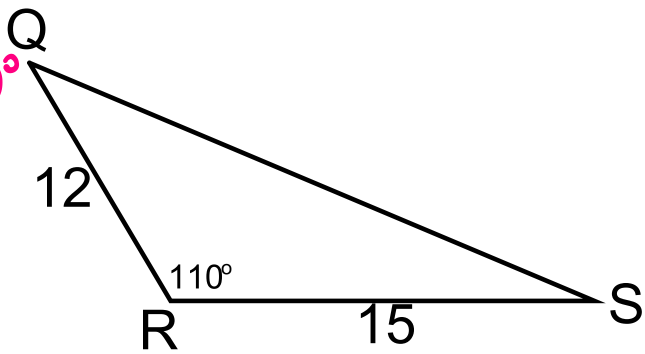
$\boxed{A = 84.6u^2}$



Proof of how the laws were created are in the slides that follow.

HW Assignment for 9.7

pg. 513:

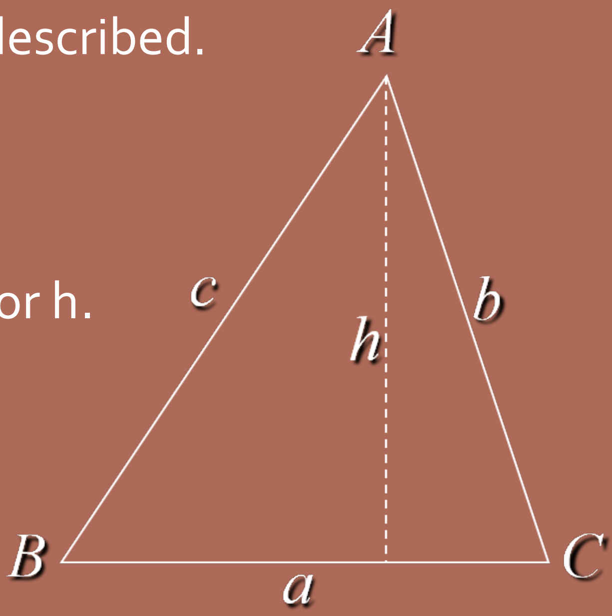
A: 7, 11, 17, 23, 25, 31, 33, 35, 37, 39, 41, 43, 55

B: 5, 7, 15, 17, 21, 23, 25, 29, 31, 33, 37, 39, 41, 55

C: 3 - 33 (o), 41, 53, 55

Proof of Sine Law

- Draw triangle ABC as described.
- Draw altitude h.
- $\sin B = {}^{h}/_{c}$
- $\sin C = {}^{h}/_{b}$
- Solve both equations for h.
- $h = c \sin B$
- $h = b \sin C$



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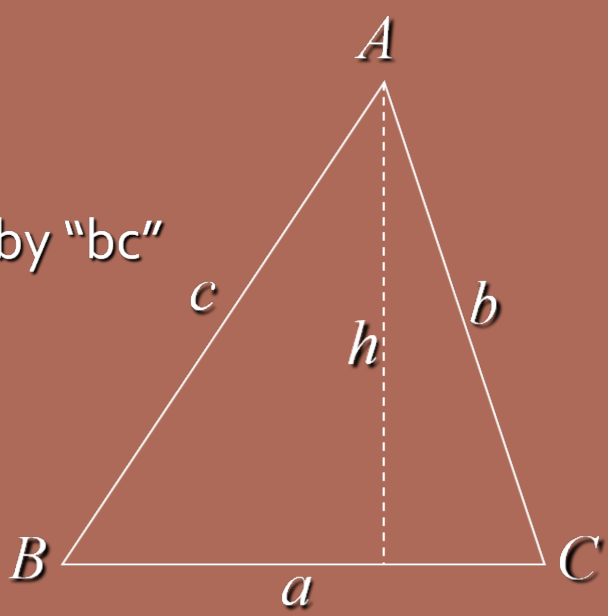
Proof of Sine Law

- $h = c \sin B$
- $h = b \sin C$
- $c \sin B = b \sin C$
- Divide both sides by "bc"
- $\dfrac{c \sin B}{bc} = \dfrac{b \sin C}{bc}$
- $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



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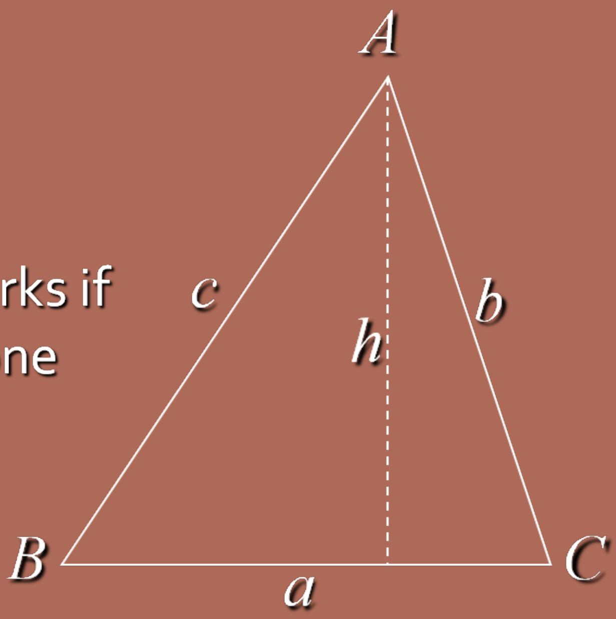
Proof of Sine Law

- Similarly, you could show
- $\frac{\sin A}{a} = \frac{\sin B}{b}$
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Note: This only works if you know at least one "angle-side" pair.



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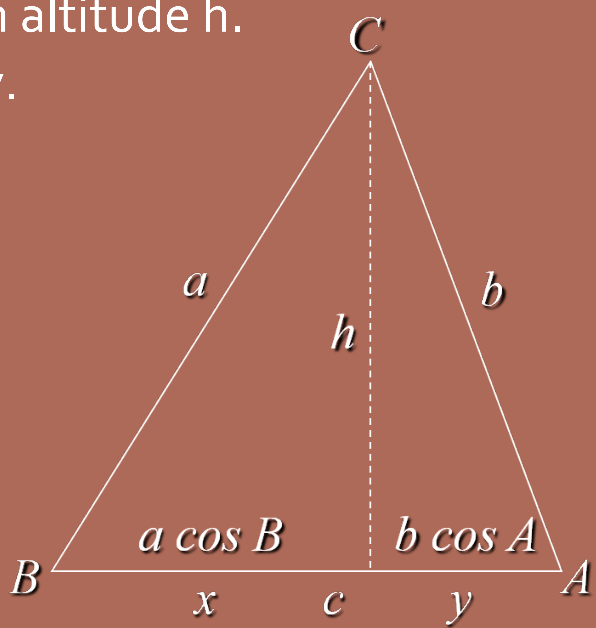
Proof of the Cosine Law

- Draw a new triangle with altitude h.
- Side c is split into x and y.
- cos B = x/a
- cos A = y/b
- x = a cos B
- y = b cos A



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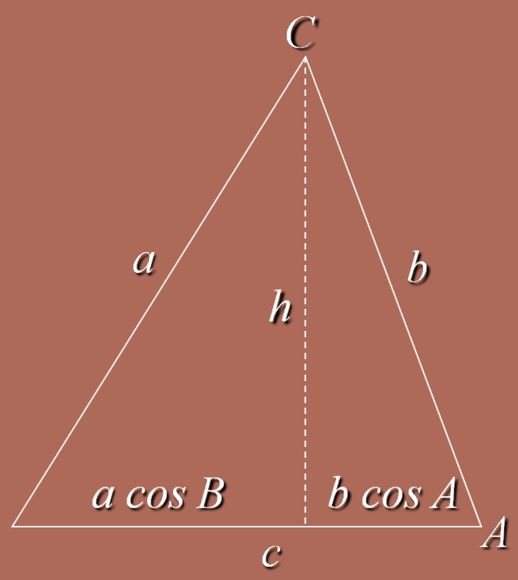
Proof of the Cosine Law

- Side c was split into two parts: "a cos B" & "b cos A".
- $c = a \cos B + b \cos A$
- Multiply by "c".
- $c^2 = ac \cos B + bc \cos A$



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Proof of the Cosine Law

- $c^2 = \underline{ac \cos B + bc \cos A}$
- Similarly, you could say...
- $a^2 = ac \cos B + ab \cos C$
- $b^2 = bc \cos A + ab \cos C$
- Add the last two equations:
- $a^2 + b^2 = ac \cos B + bc \cos A + 2\,ab \cos C$
- Rearrange:
- $\underline{ac \cos B + bc \cos A = a^2 + b^2 - 2ab \cos C}$
- $c^2 = a^2 + b^2 - 2ab \cos C$

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Proof of the Cosine Law

- $c^2 = a^2 + b^2 - 2ab \cos C$
- Similarly,
- $b^2 = a^2 + c^2 - 2ac \cos B$
- $a^2 = b^2 + c^2 - 2bc \cos A$

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