Algebra 2 Notes

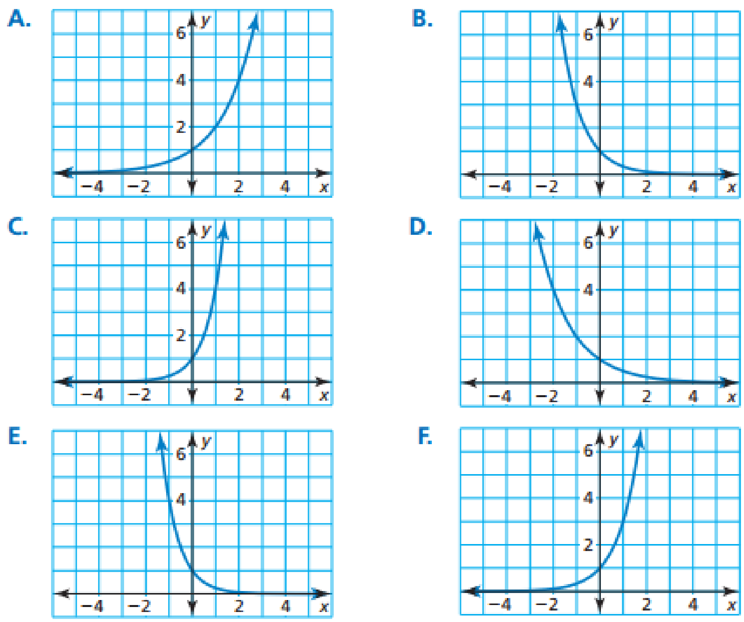
Chapter 6 Exponential & Logarithmic Functions

6.1 Exponential Growth and Decay Functions

With your table complete the following matching problems. Recall your transformations to help you. Try without a calculator. Check with a calculator!

a. $f(x) = 2^x$ **b.** $f(x) = 3^x$ **c.** $f(x) = 4^x$

d. $f(x) = \left(\frac{1}{2}\right)^x$ **e.** $f(x) = \left(\frac{1}{3}\right)^x$ **f.** $f(x) = \left(\frac{1}{4}\right)^x$



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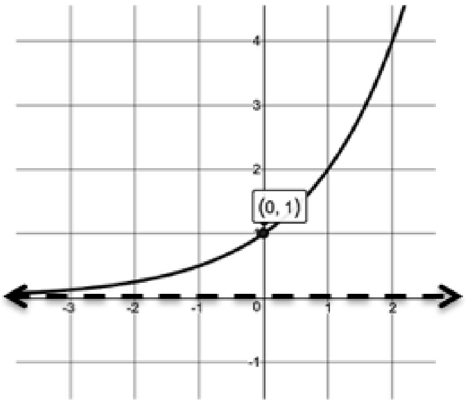
Exponential GROWTH

$y = 1(2)^x$

initial value: a =

growth factor: b =

asymptote: y =



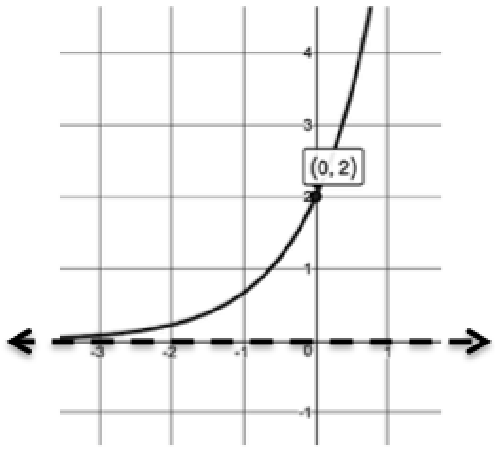
Domain: Range:

$y = 2(3)^x$

initial value: a =

growth factor: b =

asymptote: y =



Domain: Range:

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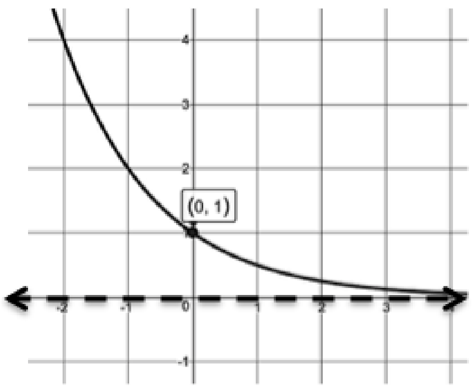
Exponential DECAY

$y = 1(\frac{1}{2})^x$

initial value: a =

decay factor: b =

asymptote: y =



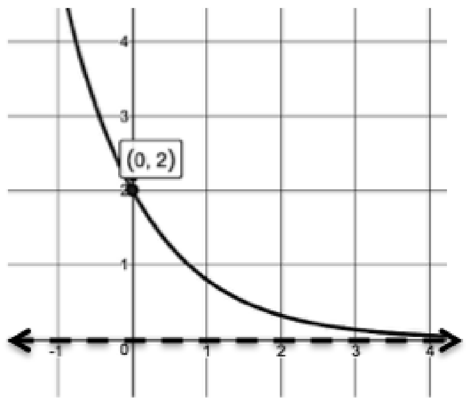
Domain: Range:

$y = 2(.4)^x$

initial value: a =

decay factor: b =

asymptote: y =



Domain: Range:

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Exponential Growth Functions

$$y = a(b)^x$$

When a > 0 and ____________________
the graph will be increasing (growing).

Exponential Decay Functions

$$y = a(b)^x$$

When a > 0 and ____________________
the graph will be decreasing (decay)

Parts of an Exponential Function

The equation for an exponential **growth function** is:
$y = a(b)^x$ where.

a: ________________________________

if b>1: ____________________________

then the graph is:

____

and r: ____________________________

The equation for an exponential **decay function** is:
$y = a(b)^x$ where

a: ________________________________

if 0<b<1: ___________________________

then the graph is:

____

and r: ____________________________

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Exponential Growth/Decay Model

$$y = a(1 \pm r)^t$$

Examples

1. $y = 3(1.8)^x$

Growth or decay

Initial value:

y-intercept:

Growth/decay factor:

Growth/decay rate:

2. $y = 2.1(1.04)^x$

Growth or decay

Initial value:

y-intercept:

Growth/decay factor:

Growth/decay rate:

3. $y = 9(.8)^x$

Growth or decay

Initial value:

y- intercept:

Growth/decay factor:

Growth/decay rate:

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Practice:

1. $y = (1.3)^x$

Growth or Decay

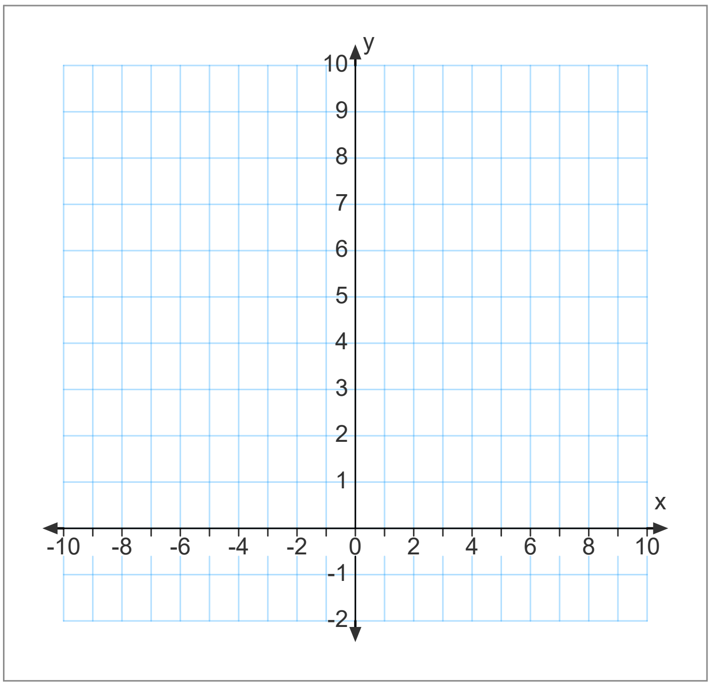
Initial Value:

y-intercept:

Growth/Decay Factor:

Growth/Decay Rate:

X	Y



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2. $y = \left(\frac{1}{3}\right)^x$

Growth or Decay

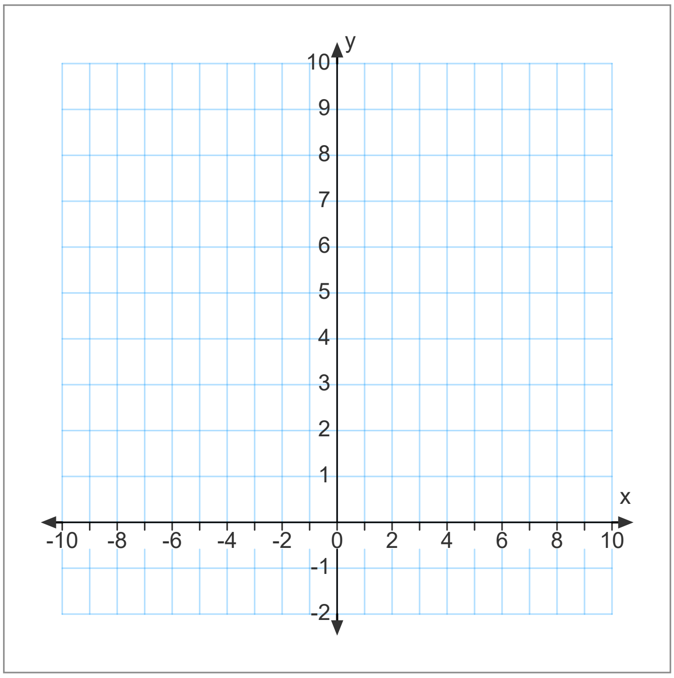
Initial Value:

y-intercept:

Growth/Decay Factor:

Growth/Decay Rate:

X	Y



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Exponential Regression

Time (seconds)	Radioactivity level
0	20
1	10
2	5
3	2.5
4	1.25

$$y = a(b)^x$$

x = time

y = Radioactivity level

x	y
-3	$y = 8 \cdot 3^{-(-3)} = 8 \cdot 3^3 = 216$
-2	$y = 8 \cdot 3^{-(-2)} = 8 \cdot 3^2 = 72$
-1	$y = 8 \cdot 3^{-(-1)} = 8 \cdot 3^1 = 24$
0	$y = 8 \cdot 3^{-(0)} = 8 \cdot 3^0 = 8$
1	$y = 8 \cdot 3^{-(1)} = 8 \cdot 3^{-1} = \frac{8}{3}$
2	$y = 8 \cdot 3^{-(2)} = 8 \cdot 3^{-2} = \frac{8}{9}$

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VOCAB!!!

Exponential equations are written in the form: ______________________________ where,

________ is the initial value/y-intercept

________ is the growth/decay factor

________ is the growth/decay rate

decay: $y = a(1 - r)^x$: $b = 1 - r$

growth: $y = a(1 + r)^x$: $b = 1 + r$

For exponential growth b > ________ : For exponential decay _____ < b < ______

Mar 22-10:01 AM

Exponential Regression Instructions:

TI Calculator:

STAT, EDIT, L1 (x's) L2 (y's)

STAT, CALC, 0 ExpReg

DESMOS:

Type in your table and use $y_1 \sim a(b)^{x_1}$

Mar 22-10:01 AM

Example 1:

x	y
0	81
1	27
2	9
3	3
4	1

Growth or decay

a: ________ b: ________

decay rate (r): ________

Exponential equation: ________________

Example 2:

x	y
1	13.2
2	29.04
3	63.888
4	140.555
5	309.22

Growth or decay

a: ________ b: ________

growth rate (r): ________

Exponential equation: ________________

Mar 22-10:02 AM

Example 3:

Year	X	Population
1972		200
1973		322
1974		519
1975		836
1976		1,346
1977		2,167
1978		3,490

Given the following data from a large mouth bass population in a local pond below, find the exponential equation that models this situation using exponential regression.
Let 0 represent the year 1970.

Growth or decay

ROUND TO 3 PLACES
a: _________ b: _________

growth rate (r): ________

Exponential equation: ____________________

Prediction:

1. Predict the population size of large mouth bass in the local pond for the year 1981.

   x = ________

   Pop. of ________________________

2. Find f(29). What year does x = 29 represent?

   Pop. of __________________________

Mar 22-10:02 AM

Compound Interest

*** rate is always a decimal!***

$$A = P\left(1+\frac{r}{t}\right)^{nt}$$

EXAMPLE:

You deposit $9000 in an account that pays 1.46% annual interest. Find the balance after 3 years when the interest is compounded quarterly.

Feb 1-9:21 AM

Practice:

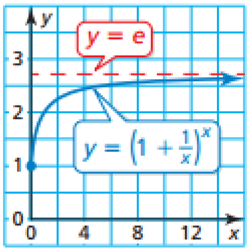
You deposit $8600 in an account that pays 1.32% annual interest. Find the balance after 4 years when the interest is compounded quarterly.

Feb 1-9:42 AM

6.2 The Natural Base e

The Natural Base *e*

The history of mathematics is marked by the discovery of special numbers, such as π and i. Another special number is denoted by the letter e. The number is called the **natural base *e***, or the *Euler number*, after its discoverer, Leonhard Euler (1707–1783). The expression $\left(1 + \frac{1}{x}\right)^x$ approaches e as x increases, as shown in the graph and table.



x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \frac{1}{x}\right)^x$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

Core Concept

The Natural Base e

The natural base e is irrational. It is defined as follows:

As x approaches $+\infty$, $\left(1 + \frac{1}{x}\right)^x$ approaches $e \approx 2.71828182846$.

6.2

Examples: Simplify the expression.

$e^6 \bullet e^3$

$\dfrac{16e^5}{4e^4}$

$\left(3e^{-4x}\right)^2$

Practice: Simplify the expression.

$e^7 \bullet e^4$

$\dfrac{24e^8}{8e^5}$

$\left(10e^{-3x}\right)^3$

Feb 14-9:12 AM

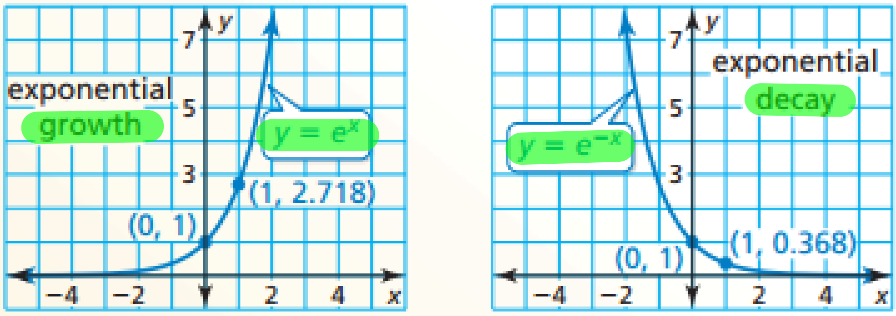
Core Concept

Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function.*

- When $a > 0$ and $r > 0$, the function is an exponential growth function.
- When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.



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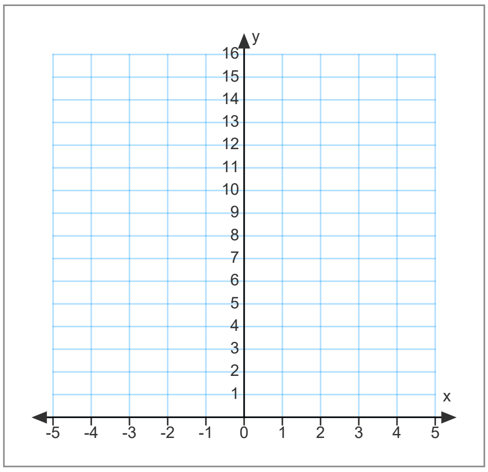
Examples: Determine whether growth of decay and then graph the function.

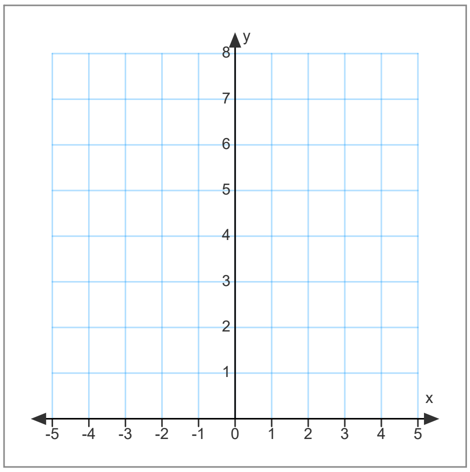
X	Y

$$y = 3e^{x}$$



$$f(x) = e^{-0.5x}$$

X	Y



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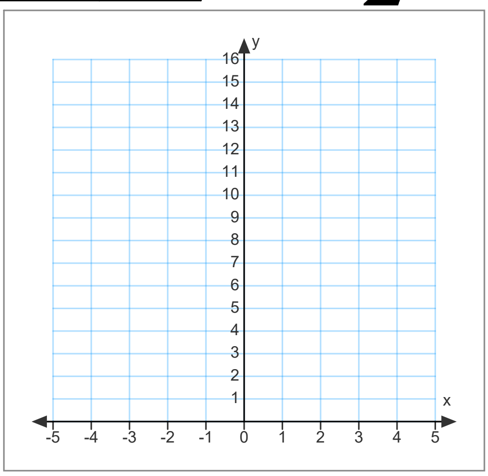
Practice: Determine whether growth of decay and then graph the function.

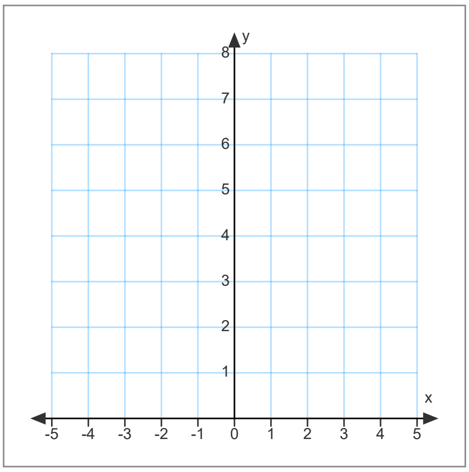
X	Y

$$y = \frac{1}{2}e^{x}$$



$$f(x) = 2e^{2x}$$

X	Y



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Continuously Compounded Interest

$$A = Pe^{rt}$$

You deposit $4250 in an account that earns 5% annual interest compounded continuously. What is the final amount after 10 years?

Feb 14-9:21 AM

Practice:

You and your friend each have accounts that earn annual interest *compounded continuously*. Your friend started with $4500 and a rate of 4%? The balance of your account after t years can be modeled by $A = 3900e^{0.05t}$. Which account has a greater principle? Which account has a greater balance after 14 years?

Feb 14-9:26 AM

6.3 Logarithms and Logarithmic Functions

Core Concept

Definition of Logarithm with Base b

Let b and y be positive real numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as

$$\log_b y = x \quad \text{if and only if} \quad b^x = y.$$

The expression $\log_b y$ is read as "log base b of y."

Since they are inverses of each other it is important to know how to switch back and forth.

There is a trick, remember "I HEART LOGS!"

$$\log_b y = x \qquad b^x = y$$

6.3

Common Logarithm: Is a logarithmic function with base 10.

$$\log_{10} x = \log x$$

Natural Logarithm: Is a logarithmic function with base "e" and is notated with an ln instead of log.

$$\log_e x = \ln x$$

Feb 20-8:18 AM

Evaluate the logarithm. If necessary, use a calculator and round your answer to 3 places.

Example:

$\log_2 32$ $\log_{27} 3$ $\log 12$ $\ln 0.75$

Practice:

$\log_4 256$ $\log_8 0.125$ $\log_{\frac{1}{3}} 27$ $\log_{64} 8$

Feb 20-7:30 AM

Using Inverse Properties to Simplify or Solve Logs!

EXAMPLE 5 **Using Inverse Properties**

Simplify (a) $10^{\log 4}$ and (b) $\log_5 25^x$.

SOLUTION

a. $10^{\log 4} = 4$ — $b^{\log_b x} = x$

b. $\log_5 25^x = \log_5 (5^2)^x$ — Express 25 as a power with base 5.

$= \log_5 5^{2x}$ — Power of a Power Property

$= 2x$ — $\log_b b^x = x$

$8^{\log_8 x}$ $\log_7 7^{-3x}$

$\log_2 64^x$ $e^{\ln 20}$

Practice:

$10^{\log 18}$ $\log_4 16^x$

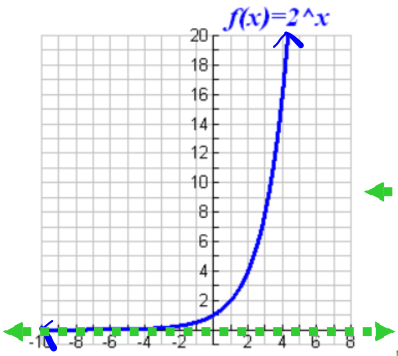
Feb 20-8:21 AM

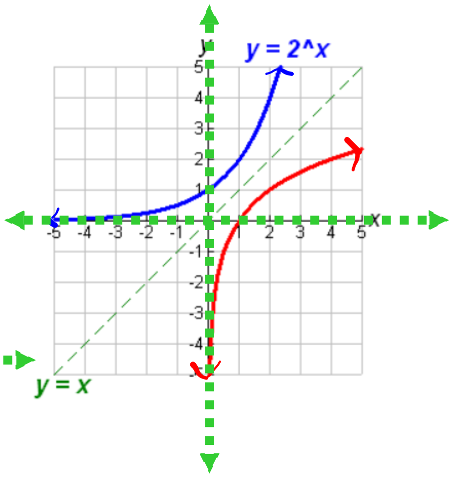
Graphing Exponential Functions and their Inverses

Most exponential graphs resemble this same shape.

$f(x) = 2^x$

x	$f(x) = y$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$





When the base is different than e or 10. Pick x values and make a T-chart.

Feb 21-7:02 AM

Change from Logarithmic form to Exponential Form

*** I ♥ LOGS***

1.) $\log_5 x = y$

2.) $\log_8 x = y$

3.) $\log_a 4 = 5$

4.) $\log_e b = -3$

Feb 14-2:05 PM

Change from Exponential to Logarithmic form:

*** I ♡ LOGS***

1.) $1.2^3 = m$

2.) $e^b = 9$

3.) $a^4 = 24$

4.) $c = 10^k$

Feb 14-2:29 PM

Evaluate the following:

1.) $\log_2 1$

2.) $\log_8 8$

3.) $\log_2 8$

4.) $\log \sqrt{10}$

5.) $\log_{\frac{1}{3}} 9$

6.) $\log 1000$

Feb 14-2:39 PM

Solve for x:

$\log_{81}\frac{1}{27}=x$

$\log_{16}64=x$

$\log_5 x=-3$

$\log_{100}x=\frac{3}{2}$

Feb 24-3:52 PM

6.4 Transformations of Exponential and Logarithmic Functions

Try to not use your calculator, instead use your transformations knowledge!

a. $g(x)=e^{x+2}-3$

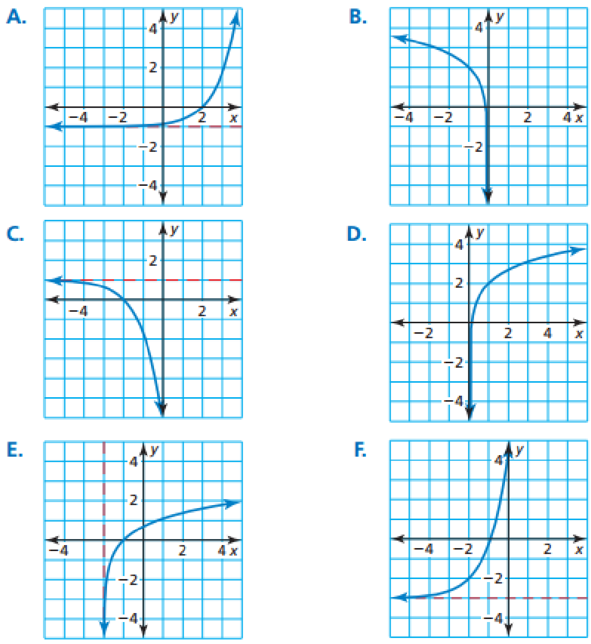
b. $g(x)=-e^{x+2}+1$

c. $g(x)=e^{x-2}-1$

d. $g(x)=\ln(x+2)$

e. $g(x)=2+\ln x$

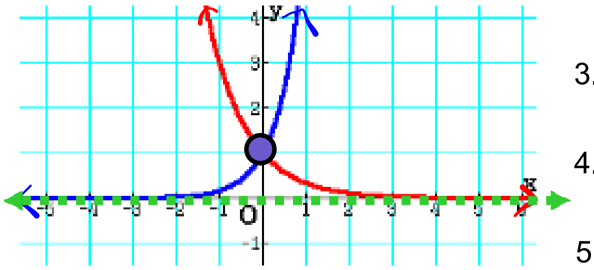
f. $g(x)=2+\ln(-x)$



Exponential Functions (Inverse of Logarithmic)

$$f(x) = 6^x$$

$$k(x) = \left(\frac{1}{3}\right)^x$$



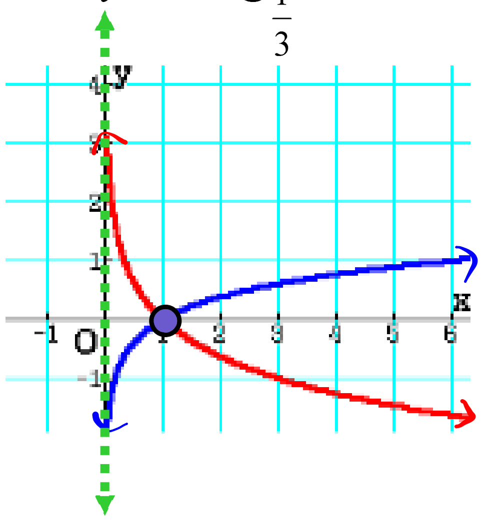
1. Base > 1, continuous, __________ curve
   0 < base < 1, continuous, __________ curve
2. Domain:
3. Range:
4. Horizontal Asymptote at __________
5. Common point (pivot point): ________

Feb 18-1:54 PM

Logarithmic Functions (Inverse of Exponential)

$$y = \log_6 x$$

$$y = \log_{\frac{1}{3}} x$$



1. Base > 1, continuous, __________ curve
   0 < base < 1, continuous, __________ curve
2. Domain:
3. Range:
4. Vertical Asymptote at __________
5. Common point (pivot point): ________

Feb 18-2:08 PM

Core Concept Exponential

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = 2^{x-3}$ $g(x) = 2^{x+2}$	3 units right 2 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = 2^x + 5$ $g(x) = 2^x - 1$	5 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = 2^{-x}$ $g(x) = -2^x$	over y-axis over x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$	$g(x) = 2^{2x}$ $g(x) = 2^{x/2}$	shrink by $\frac{1}{2}$ stretch by 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \cdot f(x)$	$g(x) = 3(2^x)$ $g(x) = \frac{1}{4}(2^x)$	stretch by 3 shrink by $\frac{1}{4}$

Feb 23-10:02 AM

Core Concept Logarithmic

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = \log(x - 4)$ $g(x) = \log(x + 7)$	4 units right 7 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = \log x + 3$ $g(x) = \log x - 1$	3 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = \log(-x)$ $g(x) = -\log x$	over y-axis over x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$	$g(x) = \log(4x)$ $g(x) = \log\left(\frac{1}{3}x\right)$	shrink by $\frac{1}{4}$ stretch by 3
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \cdot f(x)$	$g(x) = 5 \log x$ $g(x) = \frac{2}{3} \log x$	stretch by 5 shrink by $\frac{2}{3}$

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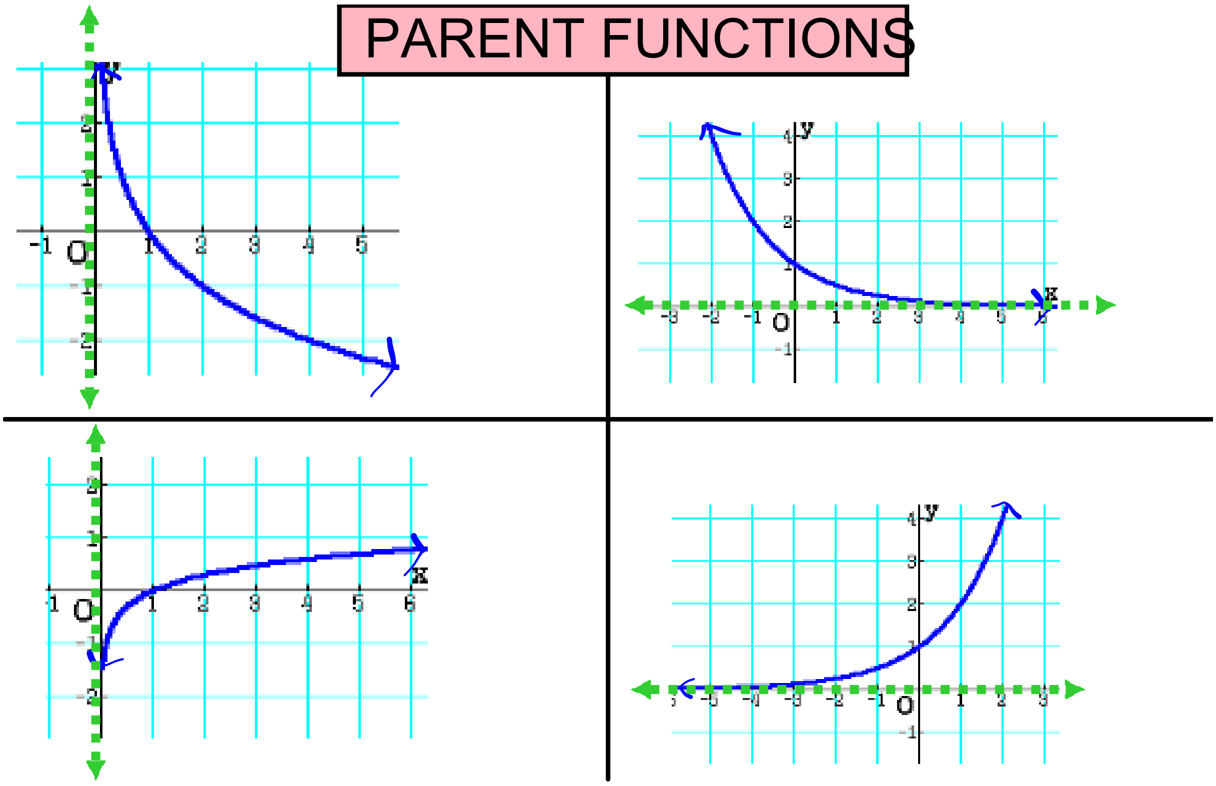
PARENT FUNCTIONS



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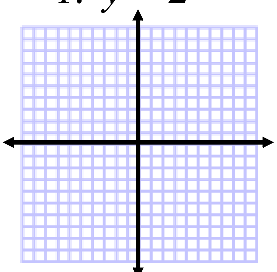
Transformations: (Always move from the parent function)

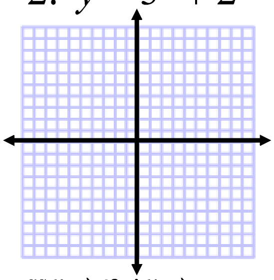
Examples: Sketch. Label the H.A. and y-intercept and pivot point. Then give the end behavior below.

1. $y = 2^{x+1}$



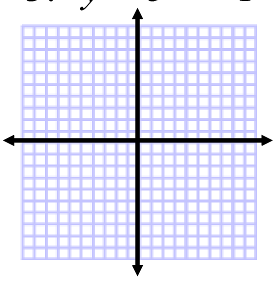
$as\ x \to \infty\ ; y \to$ _____
$as\ x \to -\infty\ ; y \to$ _____

2. $y = 3^x + 2$



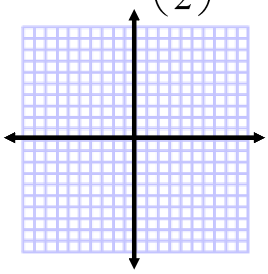
$as\ x \to \infty\ ; y \to$ _____
$as\ x \to -\infty\ ; y \to$ _____

3. $y = 5^{-x} - 1$



$as\ x \to \infty\ ; y \to$ _____
$as\ x \to -\infty\ ; y \to$ _____

4. $y = -\left(\frac{1}{2}\right)^x - 2$

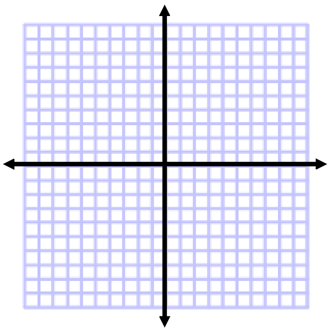


$as\ x \to \infty\ ; y \to$ _____
$as\ x \to -\infty\ ; y \to$ _____

Feb 18-2:33 PM

LOG Transformations:

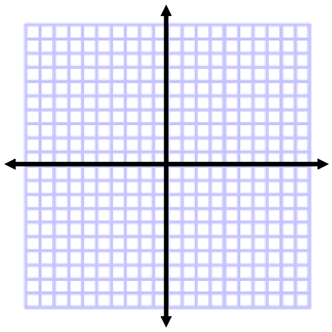
1. $y = \log_{10}(-x) + 3$



As x → 0, y → _____

As x → - ∞, y → _____

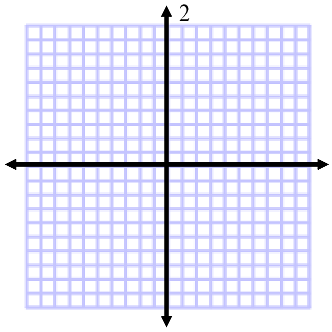
2. $y = -\ln_e(x-2)$



As x → ∞, y → _____

As x → 2, y → _____

3. $y = \log_{\frac{1}{2}}(x-3) + 1$



As x → ∞, y → _____

As x → 3, y → _____

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6.5 Properties of Logarithms

*** Reminder Exponential's and Logarithms are INVERSES!!!

So... exponent properties still apply.

Simplify.

$\left(-3x^{-2}\right)^3$

$\dfrac{81a^7}{36a^{-2}}$

$\left(2b^0c^3\right)\left(-5b^{-2}c^6\right)$

Mar 12-1:53 PM

Example: $\log_6 5 \approx 0.898$ $\log_6 8 \approx 1.161$

1. $\log_6 \frac{5}{8}$

2. $\log_6 40$

3. $\log_6 64$

4. $\log_6 125$

*** CRF: Calculator Ready Form***

Mar 12-2:20 PM

Practice: $\log_3 4 \approx 1.262$ $\log_3 5 \approx 1.465$

a. $\log_3 \frac{4}{5}$

b. $\log_3 20$

c. $\log_3 25$

Mar 12-2:22 PM

Expanding VERSUS Condensing

Expand: $\log_6 3x^4$ $\ln \frac{5}{12x}$

Condense: $\log x - \log 9$ $\ln 4 + 3\ln 3 - \ln 12$

Mar 12-2:24 PM

Practice:

Expand:

$\ln \frac{3x^5}{y}$

Condense:

$\log 6 + 4\log 3 - \log 3$

Mar 12-2:28 PM

CHANGE OF BASE FORMULA

$$\log_c a = \frac{\log_b a}{\log_b c}$$

$b \neq 1$ & $c \neq 1$

Examples: change to common log and natural log

$\log_3 8$ $\log_6 24$

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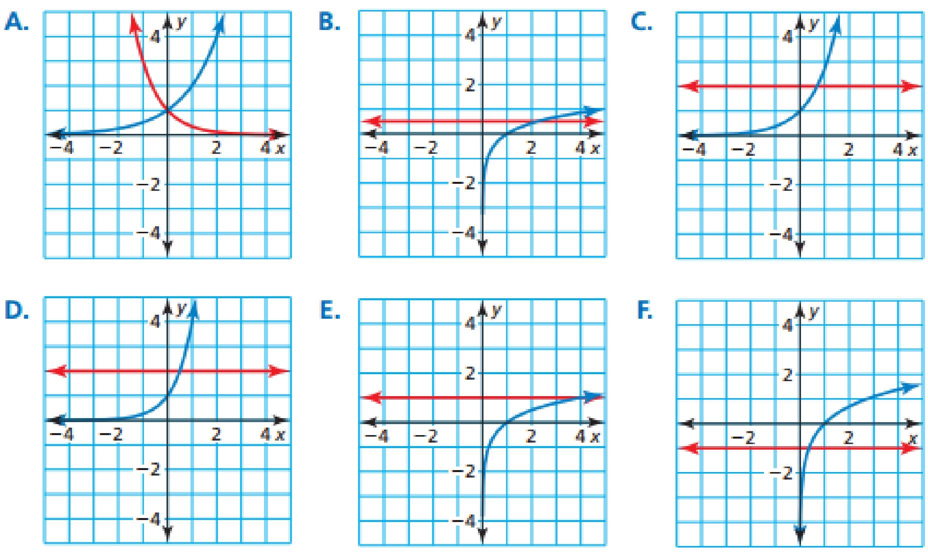
6.6 Solving Exponential and Logarithmic Equations

a. $e^x = 2$ **b.** $\ln x = -1$

c. $2^x = 3^{-x}$ **d.** $\log_4 x = 1$

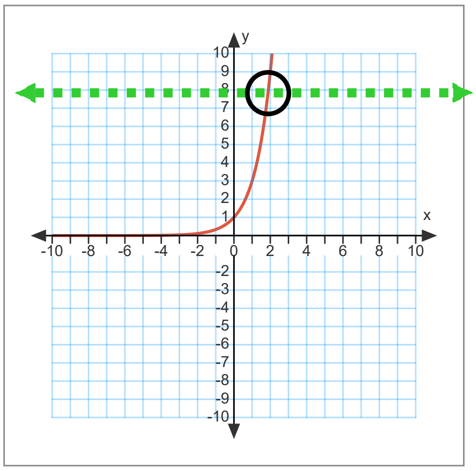
e. $\log_5 x = \frac{1}{2}$ **f.** $4^x = 2$



You can solve like a system and graph to find their intersection OR you can use the inverse to solve.

$$3^x = 8$$

$y = 3^x$ $\quad$ $y = 8$



$$3^x = 8$$

1) log both sides

2) Use properties to get x alone

3) check for extraneous solutions

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Now we know how to solve like bases and unlike bases using the change of base formula.

like bases:

$$100^x = \left(\frac{1}{10}\right)^{x+3}$$

unlike bases:

$$2^x = 5$$

I ❤ logs!

Mar 14-2:28 PM

Practice: CRF

a) $7^{9x} = 15$

b) $4e^{-x} - 7 = 13$

Mar 14-2:33 PM

Examples:

1. $\ln(7x-4) = \ln(2x+11)$

2. $\log 5x + \log(x-1) = 2$

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Practice:

a. $\log_2(x-6)=5$

b. $\log_4(x+12)+\log_4 x=3$

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An inequality is treated like an equal sign, but now there is a range of solutions rather than just one.

$10^{2x-6}>3$

$\log x+9<45$

$2\ln x-1>4$

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6.7 Modeling with Exponential & Logarithmic Functions

Year,X	# of hoops, Y
1	17
2	31
3	50
4	85
5	156
6	274
7	498
10	

Using Exponential Regression we will write the equation given coordinates or a graph.

a)Write an exponential equation for the data provided.

b) Predict for 10 years using your equation.

6.7

Example: Write an equation and predict.

X	Y
-2	243
-1	81
0	27
1	9
2	3
3	1

equation:

____

Predict for -8.

Mar 15-12:36 PM

Practice: Write an equation and predict.

Year,X	# of trampolines, Y
1	15
2	23
3	40
4	52
5	80
6	105
7	140

equation:

____

How many trampolines will there be after 20 years?

After how many years will there be 250?

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